

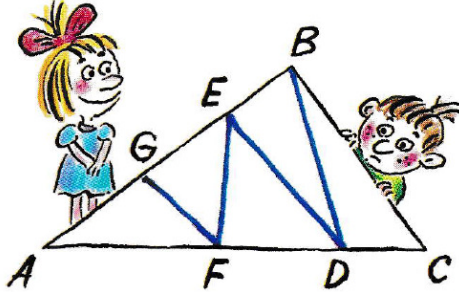
Equitable Slice Problem

18 March 2021

Jim Stevenson

This is another Brainteaser from the *Quantum* math magazine ([1]).

How can a polygonal line BDEFG be drawn in a triangle ABC so that the five triangles obtained have the same area?



Art by Edward Nazarov

however, was tantamount to just saying divide the triangle into triangles of equal area—without providing a method! That is, no solution at all.

My Solution

My solution depends heavily on the area-preserving properties of reflections and the “shear” action on triangles. Figure 1 through Figure 4 show the steps to solving the problem. The first triangle uses the altitude of the big triangle and base $1/5$ of AC, thus giving it an area of $1/5$ of the large triangle (Figure 1). Subsequent equal triangles are constructed via reflections and shearing, thus preserving the $1/5$ area of the larger triangle.

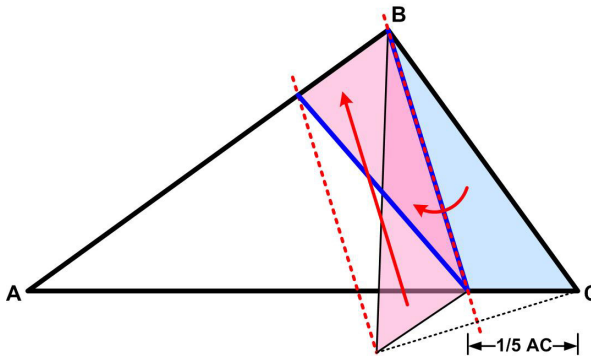


Figure 1 Lay out first triangle with base = $1/5$ AC. Flip it around its side and then shear to the side of the large triangle.

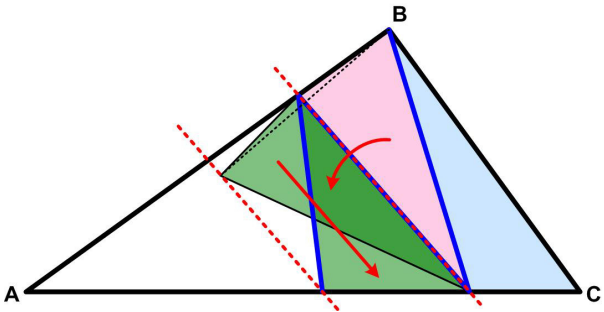


Figure 2 Flip second triangle around its side and shear result down to the base of the large triangle.

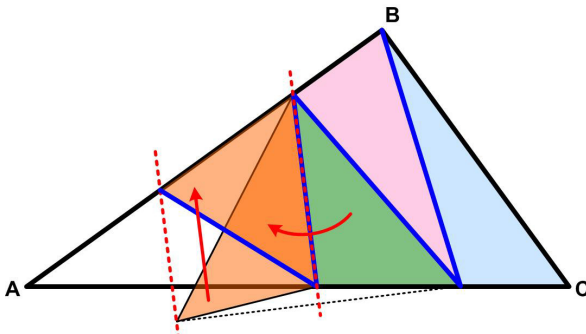


Figure 3 Flip third triangle around its side and shear result up to the side of the large triangle.

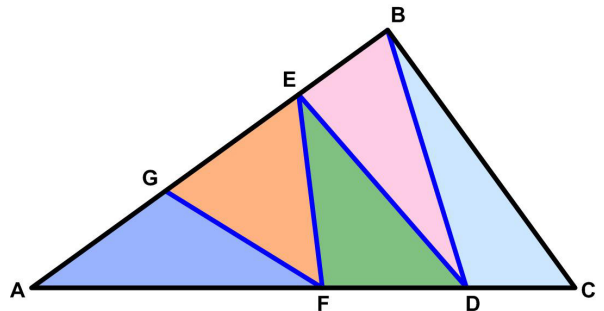


Figure 4 Remaining space ($1 - 4/5 = 1/5$) constitutes the fifth triangle of equal area.

Quantum Solution

Here is the *Quantum* solution, which essentially amounts to saying divide the triangle into triangles of equal area. That is, it restates the problem. No method is offered as to how to do it, so I don't consider it a real solution.

Point D should be positioned so that segment CD is equal to $\frac{1}{5}$ of segment AC (Figure 5); then the area of triangle DBC will be $\frac{1}{5}$ that of ABC. Similarly, point E is positioned so that $BE = \frac{AB}{4}$, point F so that $FD = \frac{AD}{3}$, and point G so that $EG = \frac{AE}{2}$.

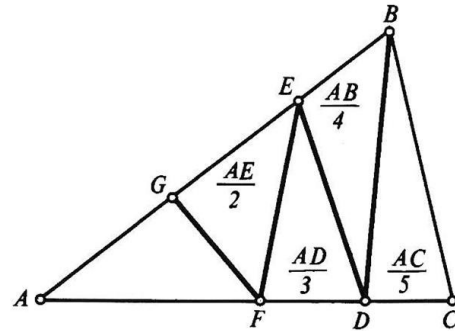


Figure 5 *Quantum Solution*

References

- [1] "Equitable Slice", B11 "Brainteasers" *Quantum* Vol.1, No.1, National Science Teachers Assoc., Springer-Verlag, Sep-Oct 1990. p.19

© 2021 James Stevenson
