Passion Kiss Problem

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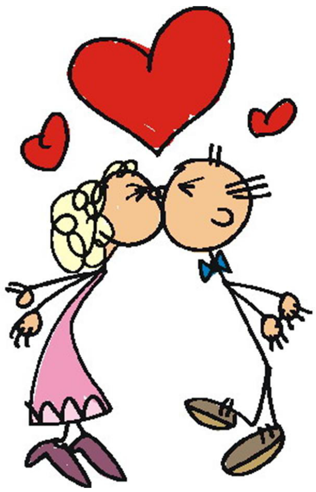

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This is a somewhat challenging math cryptogram in a slightly different guise from the 1979 Canadian Math Society's magazine, *Crux Mathematicorum* ([1]).

"But you can't make arithmetic out of passion. Passion has no square root." (Steve Shagan, *City of Angels,* G.P. Putnam's Sons, New York, 1975, p. 16.)

On the contrary, show that in the decimal system

$$\sqrt{PASSION} = KISS$$

has a unique solution.

My Solution

The first thing we do is recognize this is a math cryptogram of the SEND-MORE-MONEY type, only as a multiplication problem instead of an addition problem, namely

KISS x KISS = PASSION

So we multiply the digits in symbolic form as shown in Table 1. We then get the following mod 10 equations for the digits in PASSION.

$P = K^2 + c$

$A =_{\text{mod } 10} 2IK + c$

$S =_{\text{mod } 10} (2SK+I^2) + c$

$S =_{\text{mod } 10} 2S(I+K) + c$

$I =_{\text{mod } 10} (2SI+S^2) + c$

$O =_{\text{mod } 10} 2S^2 + c$

$N =_{\text{mod } 10} S^2$

where c is the value carried from the previous digit to the right.

Table 1 Original Product KISS² = PASSION

			K	**I**	**S**	**S**
			K	**I**	**S**	**S**
			SK	SI	S^2	S^2
		SK	SI	S^2	S^2	
	IK	I^2	IS	IS		
K^2	KI	KS	KS			
K^2	2IK	$2SK+I^2$	2S(I+K)	$2SI+S^2$	$2S^2$	S^2
P	**A**	**S**	**S**	**I**	**O**	**N**

Now since $P = K^2 + c$ must be a single digit, K = 1, 2, or 3. We proceed by trial and error to compute the digits in PASSION, starting with the right-most digit N, as shown in Table 2 We consider values of S = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and the corresponding possibilities for N, O, etc., making sure that no letter-digit correspondences are duplicated. The challenge, of course, is to harness this nightmare in bookkeeping into some type of organized pattern. Nevertheless, I made a slew of mistakes and omissions until I got things under control.

Table 2 Trial and Error Computations for digits in PASSION, Proceeding from Right to Left

	P	A	S	S	I	O	N	K
	K^2 + c	2IK + c	$2SK+I^2$ + c	2S(I+K) + c	$2SI+S^2$ + c	$2S^2$ + c	S^2	
S = 0 ⇒ N = 0 (S ≠ N) X			0?	0?			X	
S = 1 ⇒ N = 1 (S ≠ N) X			1?	1?			X	
S = 2 ⇒ **N = 4** ⇒ **O = 8** (carry 0)			2?	2?		8?	4?	
I = $2SI+S^2+c$ ⇒ I = 4I+4+0 ⇒ I even, but								
I ≠ 0, ≠ 6 X					X			
S = 3 ⇒ **N = 9** ⇒ O = 2·9 = 18 ⇒			3?	3?			9?	
O = 8 (carry 1)						8?		
I = $2SI+S^2+c$ ⇒ I = 6I+9+1 ⇒ I even,								
I = 0? = 2? = 4? = 6?								
I = 0 ⇒ 0 = 6·0+9+1 = 10✓ ⇒ **I = 0** (carry 1)					0?			
K = 1 ⇒ 3 = 6(0+1)+1 = 7[1] X								X
K = 2 ⇒ 3 = 6(0+2)+1 = 13✓ (S, carry 1)								
3 = 6·2+0^2+1 = 13[2]✓ (S, carry 1)								2?
A = 2·0·2+1 = 1 ⇒ **A = 1**		1?						
P = 4 (a solution)	**4**	**1**	**3**	**3**	**0**	**8**	**9**	**2**
I = 2 ⇒ 2 = 6·2+9+1 = 22✓ ⇒ **I = 2** (carry 2)					2?			
K = 1 ⇒ 3 = 6(2+1)+2 = 20 X								X
I = 4 ⇒ 4 = 6·4+9+1 = 34✓ ⇒ **I = 4** (carry 3)					4?			
K = 1 ⇒ 3 = 6(4+1)+3 = 33✓ (S, carry 3)								
3 = 6·1+4^2+3 = 25 X								X
K = 2 ⇒ 3 = 6(4+2)+3 = 39 X								X
I = 6 ⇒ 6 = 6·6+9+1 = 46✓ ⇒ **I = 6** (carry 4)					6?			
K = 1 ⇒ 3 = 6(6+1)+4 = 46 X								X
K = 2 ⇒ 3 = 6(6+2)+4 = 52 X								X
S = 4 ⇒ N = 16 ⇒ **N = 6** (carry 1)			4?	4?			6?	
⇒ O = 2·16+1 = 33 ⇒ **O = 3** (carry 3)						3?		
I = $2SI+S^2+c$ ⇒ I = 8I+16+3 ⇒ I odd, but								
I ≠ 1, ≠ 5, ≠ 7, ≠ 9					X			
S = 5 ⇒ N = 25 ⇒ **N = 5** (carry 2) X			5?	5?			X	
S = 6 ⇒ N = 36 ⇒ **N = 6** (carry 3) X			6?	6?			X	

[1] S $=_{\text{mod } 10}$ 2S(I+K) + c
[2] S $=_{\text{mod } 10}$ $2SK+I^2$ + c

	P	A	S	S	I	O	N	K
	K^2 + c	2IK + c	$2SK+I^2$ + c	2S(I+K) + c	$2SI+S^2$ + c	$2S^2$ + c	S^2	
S = 7 ⇒ N = 49 ⇒ **N = 9** (carry 4) ⇒ O = 2·49+4 = 102 ⇒ **O = 2** (carry 10) $I = 2SI+S^2+c$ ⇒ I = 14I+49+10 ⇒ I odd, but I ≠ 1, ≠ 3, ≠ 5 X			7?	7?	X	2?	9?	
S = 8 ⇒ N = 64 ⇒ **N = 4** (carry 6) ⇒ O = 2·64+6 = 134 ⇒ **O = 4** (carry 13) X			8?	8?		X	4?	
S = 9 ⇒ N = 81 ⇒ **N = 1** (carry 8) ⇒ O = 2·81+8 = 170 ⇒ **O = 0** (carry 17) $I = 2SI+S^2+c$ ⇒ I = 18I+81+17 ⇒ I even, I ≠ 2, ≠ 4, = 6? ≠ 8 I = 6 ⇒ 6 = 18·6+81+17 = 206✓ ⇒ **I = 6** (carry 20) K = 2 ⇒ 9 = 18(6+2)+20 = 164[3] X K = 3 ⇒ 9 = 18(6+3)+20 = 182 X			9?	9?	6?	0?	1?	X X

Therefore KISS = 2033 and PASSION = 4,133,089 is the *unique* solution.

Check: KISS x KISS = PASSION?

2033^2 = 4,133,089 ✓

It should be noted that we essentially only had to consider 29 cases.

Crux Mathematicorum Solutions

I. *Voyeuristic solution and clinical report by Clayton W, Dodge, University of Maine at Orono* [with *riposte* by Edith Orr]. ([2])

In view of the number of digits in PASSION [that's it, keep your hands right there where I can see them], we must have 1000 < KISS < 3162. Since S ≠ N, we must also have S ≠ 0, l, 5, 6. Now all we have to do is to square each KISS with K = l, 2, 3 and S = 2, 3, 4, 7, 8, 9. [JOS: See following comment.] [Yes, we could program a calculator to do it, but what's your hurry?] As each KISS is squared [you'll have to pucker your lips, dear], "look for the double S in PASSION. A near solution is 2877^2 = 8277129. [Let's try again, shall we?] Finally [ahhh, all PASSION spent at last], we are left with the unique solution

$$\sqrt{4133089} = 2033.$$

Clinical report. Observe that 2033 is not prime but is the product of the two primes 19 and 107, representing the union of two individuals, each with its own uniqueness. Alas, their PASSION is neither perfect nor abundant, but sadly deficient. Send them to Masters and Johnson.

Comment. What about I? Do we try all I = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, together with the limited choices for K and S, making sure I ≠ K ≠ S? If so, Table 3 shows that the number of multiplications is 128. That's a lot of trial and error. As I indicated above, I only had to consider basically 29

[3] $S =_{\text{mod } 10} 2S(I+K) + c$

Table 3 Number of $KISS^2$ to Compute for $K \neq I \neq S$

K = 1,2,3	I = 0,1,2,3,4,5,6,7,8,9	S = 2,3,4,7,8,9		
1	3 choices {0,5,6}	6 choices	18 choices	
	6 choices {2,3,4,7,8,9}	5 choices	30 choices	48 choices
2	4 choices {0,1,5,6}	5 choices	20 choices	
	5 choices {3,4,7,8,9}	4 choices	20 choices	40 choices
3	4 choices {0,1,5,6}	5 choices	20 choices	
	5 choices {2,4,7,8,9}	4 choices	20 choices	40 choices
				128 choices

choices and the multiplications involved were largely simpler than computing $KISS^2$ by hand, which is very inelegant.

II. *Solution by Allan Wm. Johnson Jr., Washington, D.C.* [with a cameo appearance by the irrepressible Edith].

As we will see below, there is a unique solution in the decimal system, and in this solution I = 0 and A = 1. But there are infinitely many other bases B in which there is at least one solution, even with I = 0 and A = l. This is shown by the identity

$$\{xB^3 + (2x - l)B + (2x - l)\}^2 = x^2B^6 + B^5 + (2x - l)B^4 + (2x - l)B^3 + (B - 2)B + (B - 1),$$

where $B = (2x - l)^2 + l$ and $x > l$. For x = 2, we obtain the decimal solution $2033^2 = 4133089$.

To show that it is unique, we can proceed as follows ... [but it's a lot more fun to do it as in solution I].

Also solved by HAYO AHLBURG, Benidorm, Spain; CECILE M. COHEN, John F. Kennedy H.S., New York, N.Y.; FRIEND H. KIERSTEAD, Jr., Cuyahoga Falls, Ohio; F.G.B. MASKELL, Algonquin College, Ottawa; J.A. McCALLUM, Medicine Hat, Alberta, LEROY F. MEYERS, Ohio State University; HERMAN NYON, Paramaribo, Surinam; CHARLES W. TRIGG, San Diego, California; KENNETH M. WILKE, Topeka, Kansas; and the proposer.

Editor's comment.

Another first for *Crux Mathematicorum:* an X-rated solution! If you don't know Edith Orr, get acquainted with her by looking up [1977: 39, 129, 224, 225; 1978: 8-10].

Comment. I have to confess I don't follow the reasoning in this second solution. The statement "As we will see below, … in this solution I = 0 and A = 1." actually seems to be *assumed* rather than *derived* in the identity presented. Furthermore, I could not successfully translate the cryptogram symbols into the coefficients of the powers of B as shown. For example, why is K = x and S = 2x–1? And what about carrying? The lack of explanation implies that it should be obvious. Sigh.

Finally, I wish the solution had continued to show its method of proving uniqueness without resorting to the "check all multiplications" in solution I.

References

[1] Wayne, Alan, "Problem 411," *Crux Mathematicorum*, Vol.5 No.2 Feb, Canadian Mathematical Society, 1979. p.46

[2] Wayne, Alan, "Problem 411 Solution," *Crux Mathematicorum*, Vol.5 No.10 Dec, Canadian Mathematical Society, 1979. p.299.