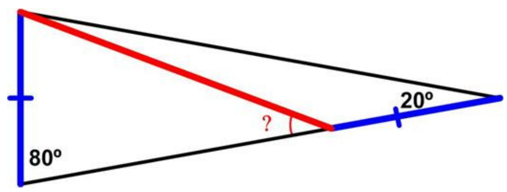
# Shy Angle Problem

6 February 2020

Jim Stevenson




Here is yet another problem from Presh Talwalkar.[1] This one is rather elegant in its simplicity of statement and answer.

**Solve For The Angle – Viral Puzzle**

24 January 2019

I thank Barry and also Akshay Dhivare from India for suggesting this problem! This puzzle is popular on social media. What is the measure of the angle denoted by a "?" in the following diagram? You have to solve it using elementary geometry (no trigonometry or other methods). I admit I did not solve it. Can you figure it out?

## My Solution

Actually I found this problem so frustrating (but enticing) that I cheated! I used Viseo to make a scaled drawing and then measured the angle. To my surprise, I found it was 30 degrees (to within reasonable error). That gave me some clues as to how to solve it. But the answer is so simple, I am sure there is an easier path than the one I found.

Knowing the answer, I set about trying to find a 30-60 right triangle somewhere. Finally, after a lot of dead ends, I found it when I tried to get the two equal blue segments close together (Figure 1). The key was to overlay a reflected image of the original triangle. I had already realized the original triangle was isosceles, since the other base angle had to be

$$180º – (80º + 20º) = 80º$$

Therefore, subtracting the original 20º from the 80º left 60º.

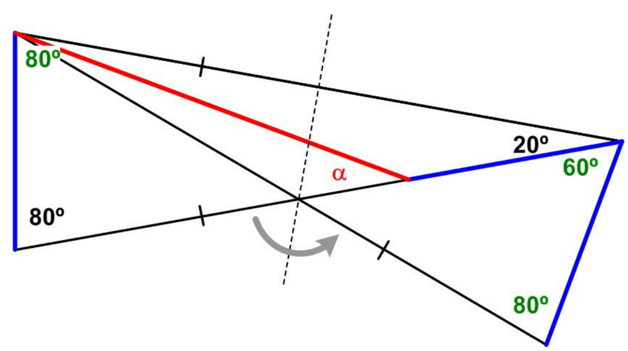



**Figure 1 Solution Step 1**

Seeing that I had equal line segments making a 60º angle, I realized I had an equilateral triangle (Figure 2). I then drew a (green) line parallel to the blue base of the reflected triangle. I wanted to show that the red line was a perpendicular bisector of the green line.

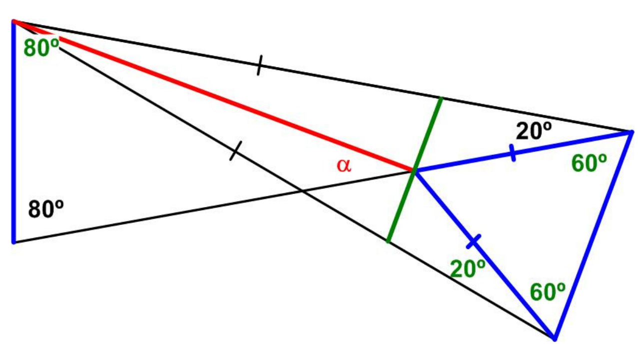



**Figure 2 Solution Step 2**

[1] https://mindyourdecisions.com/blog/2019/01/24/solve-for-the-angle-viral-puzzle/

As Figure 3 indicates, the green line being parallel to the blue base line means it makes angles of 60º with the sides of the equilateral triangle. So the resulting 20-60 triangles are similar, and in fact congruent, since one pair of their corresponding sides are equal. That means the vertex of the equilateral triangle divides the green line into two equal segments. Thus the two triangles sharing the red line are also congruent, having corresponding equal sides (the green-based triangle is similar to the blue-based isosceles triangle, and so is isosceles as well). Therefore the two angles the red line makes with the green line are equal and must add to 180º, making them 90º each.

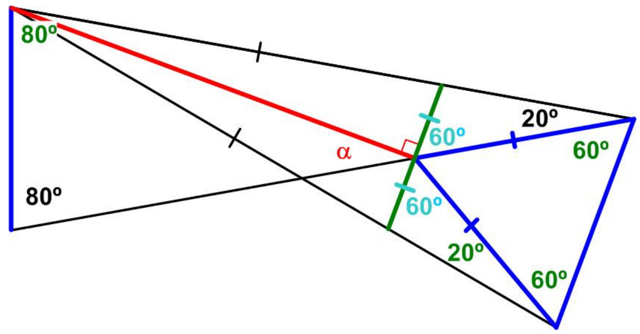



**Figure 3 Solution Step 3**

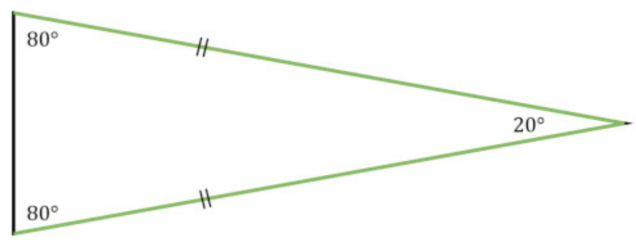
So we finally have

$$180^\circ = \alpha + 90^\circ + 60^\circ \;\Rightarrow\; \alpha = 30^\circ$$

## Talwalkar's Solution

I suppose it is a toss-up which solution is simpler. It is interesting that both rely on an equilateral triangle to introduce the needed 60º.

We will solve the problem in several steps. First, we solve for the third angle in the large triangle.



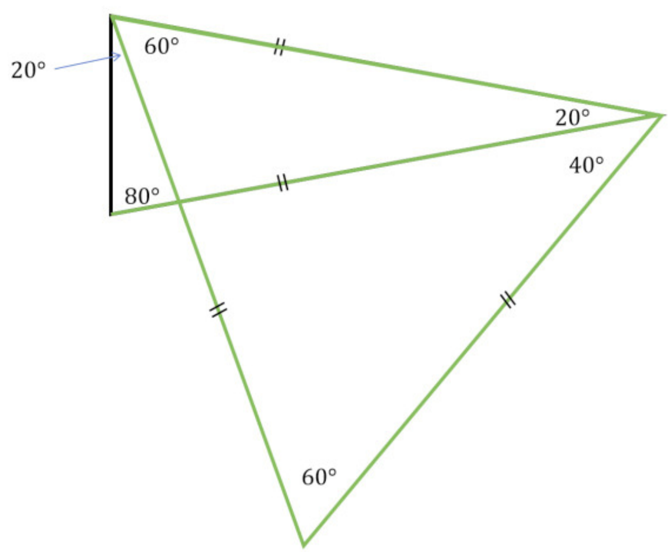

The third angle is

180 – 20 – 80 = 80 degrees,

so this is an isosceles triangle. The two sides opposite the 80 degree angles are equal.

Now comes the magical trick. Construct an equilateral triangle along the top side as follows:

Since the equilateral triangle has 60 degree angles, we can solve for the three angles as shown. Then, since the large triangle has an 80 degree angle in the upper left, and 60 degrees go to the equilateral triangle, the remaining angle must be 20 degrees as shown.

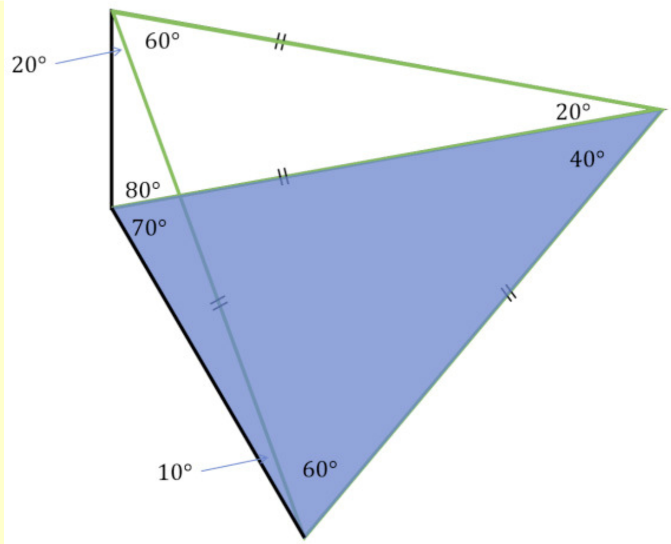


Now we will create a triangle by connecting the lower vertex of the large triangle to the lower vertex of the equilateral triangle.

The shaded blue triangle has two sides of equal length, so it is an isosceles triangle with a central angle of 40 degrees. The other two angles are then equal to

(180 – 40)/2 = 70 degrees.

So we can write 70 in one corner and then 10 in the other.




Now remove the equilateral triangle and focus on the triangles shaded in purple.

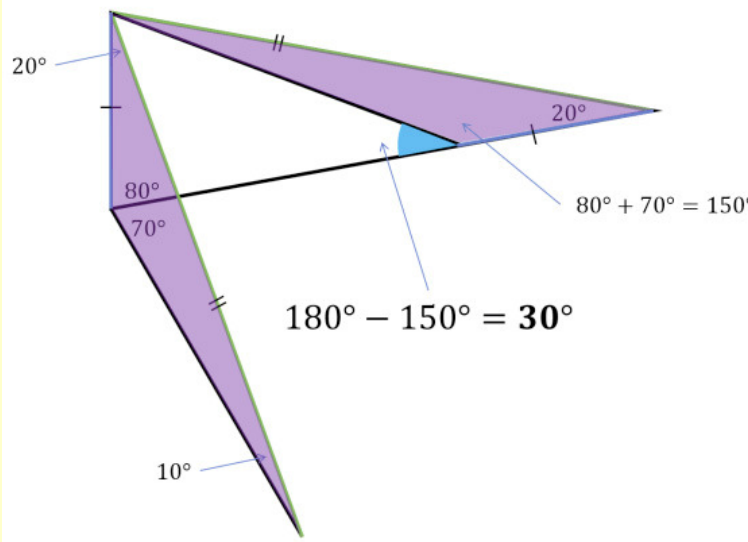
The two triangles are congruent by side-angle-side, since the two blue sides (one hash mark) are given to be equal, then there is a 20 degree angle in both, and the adjacent green sides (two hash marks) are also equal.

Thus the obtuse angles are equal, and the obtuse angle equals

80 + 70 = 150 degrees.

The desired angle is then supplementary to this, so it equals

180 – 150 = 30 degrees.




The problem is not too impossible, but it requires a fairly involved construction. Using computer graphics it was easy to illustrate each step too—but imagine trying to draw this by hand and explain it on a chalkboard! Many, like me, would get lost in an alphabet soup trying to keep all the triangles and congruent sides in order!

## Sources

Barry and Akshay Dhivare from India suggested the problem to me via email.

I found the solution method on these pages:

https://gogeometry.blogspot.com/2008/05/elearn-geometry-problem-10.html

https://www.youtube.com/watch?v=7ECGjyk5zb8