Do I Avoid Kangaroos?

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This is a fun logic puzzle from one of Ian Stewart's many math collections ([1]). I discovered that the problem actually is basically one of Lewis Carroll's examples from an 1896 book ([2]).

1. The only animals in this house are cats.
2. Every animal that loves to gaze at the moon is suitable for a pet.
3. When I detest an animal, I avoid it.
4. No animals are meat-eaters, unless they prowl by night.
5. No cat fails to kill mice.
6. No animals ever take to me, except those in this house.
7. Kangaroos are not suitable for pets.
8. Only meat-eaters kill mice.
9. I detest animals that do not take to me.
10. Animals that prowl at night love to gaze at the moon.

If all these statements are correct, do I avoid kangaroos, or not?

My Solution

Since the problem is actually an example in Carroll's *Symbolic Logic* book, it can be solved with symbolic logic and that is how Stewart does it, as shown below. But I think the problem is sufficiently straight-forward that we can retain the text—after some special reformatting.

We wish to couch the 10 statements in an "if …, then …" format, that is, in the form of implications $P \Rightarrow Q$, where P and Q are sub-statements. The following transformation of the 10 statements shows what I mean.

1. If an animal is in this house, then the animal is a cat.
2. If an animal loves to gaze at the moon, then the animal is suitable for a pet.
3. If I detest an animal, then I avoid the animal.
4. If an animal is a meat-eater, then the animal prowls by night
5. If an animal is a cat, then the animal kills mice
6. If an animal takes to me, then the animal is in this house.
7. If an animal is a Kangaroo, then the animal is not suitable for a pet.
8. If an animal kills mice, then the animal is a meat-eater
9. If an animal does not take to me, then I detest the animal.
10. If an animal prowls by night, then the animal loves to gaze at the moon.

Now there are a couple of logic rules that will help in our reasoning, which I will couch in symbolic logic form. (Note that $\sim A$ is the logical negation (or "not") of A, that is, if A is true, then $\sim A$ is false, and if A is false, then $\sim A$ is true.)

(Transitive Rule)	If A, B, and C are three statements, and if A $\Rightarrow$ B and B $\Rightarrow$ C, then A $\Rightarrow$ C
(Contrapositive)	If A and B are two statements, and if A $\Rightarrow$ B, then ~B $\Rightarrow$ ~A, which latter is called the *contrapositive*.
(Converse)	If A and B are two statements, and if A $\Rightarrow$ B, then the implication B $\Rightarrow$ A is called the *converse* (and is *only* true of A and B are logically equivalent, that is, A is true if and only if B is true)

In words, the contrapositive statement is if A is true means B is true, then if B is not true, then A must not be true either. Be careful not to confuse the contrapositive with the converse, which is not always true.

Now, reorder the 10 statements in "A $\Rightarrow$ B and B $\Rightarrow$ C and C $\Rightarrow$ D …" chains as best as possible.

9. If an animal does not take to me, then I detest the animal.
3. If I detest an animal, then I avoid the animal.

6. If an animal takes to me, then the animal is in this house.
1. If an animal is in this house, then the animal is a cat.
5. If an animal is a cat, then the animal kills mice
8. If an animal kills mice, then the animal is a meat-eater
4. If an animal is a meat-eater, then the animal prowls by night
10. If an animal prowls by night, then the animal loves to gaze at the moon.
2. If an animal loves to gaze at the moon, then the animal is suitable for a pet.

7. If an animal is a Kangaroo, then the animal is not suitable for a pet.

Then, using the transitive rule we can reduce the list of statements to three:

93. If an animal does not take to me, then I avoid the animal.

61584102. If an animal takes to me, then the animal is suitable for a pet.

7. If an animal is a Kangaroo, then the animal is not suitable for a pet.

If we take the contrapositive of the middle statement 61584102, then we have

7. If an animal is a Kangaroo, then the animal is not suitable for a pet.

61584102. If the animal is not suitable for a pet, then the animal does not take to me.

93. If an animal does not take to me, then I avoid the animal.

Applying the transitive rule again yields the final answer

If an animal is a Kangaroo, then I avoid the animal.

So, yes, I avoid kangaroos.

Clearly, the hard part of the problem is the translation of the original statements into the form of implications P $\Rightarrow$ Q.

Professor Stewart's Solution

I avoid kangaroos.

Write the conditions symbolically, as on page 275 [of ([1])]. Let

A = avoided by me

C = cat

D = detested by me

E = eats meat

H = in this house

K = kangaroos

L = loves to gaze at the moon

M = kills mice

P = prowls by night

S = suitable for pets

T = takes to me

Then with $\Rightarrow$ meaning 'implies' and $\neg$ meaning 'not', the statements (in order) become

$$H \Rightarrow C,\ L \Rightarrow S,\ D \Rightarrow A,\ E \Rightarrow P,\ C \Rightarrow M,\ T \Rightarrow H,\ K \Rightarrow \neg S,\ M \Rightarrow E,\ \neg T \Rightarrow D,\ P \Rightarrow L$$

Now we appeal to the laws of logic that I mentioned on page 275 [of ([1])]:

$$X \Rightarrow Y \text{ is the same as } \neg Y \Rightarrow \neg X$$

$$\text{If } X \Rightarrow Y \Rightarrow Z, \text{ then } X \Rightarrow Z$$

Using these laws, we can rewrite these conditions as

$$\neg A \Rightarrow \neg D \Rightarrow T \Rightarrow H \Rightarrow C \Rightarrow M \Rightarrow E \Rightarrow P \Rightarrow L \Rightarrow S \Rightarrow \neg K$$

so that $\neg A \Rightarrow \neg K$, or equivalently, $K \Rightarrow A$.

Therefore I avoid kangaroos.

References

[1] Stewart, Ian, "Do I Avoid Kangaroos?" *Professor Stewart's Hoard of Mathematical Treasures*, Profile Books Ltd., London, 2009 (Basic Books, 2010). p.180

[2] Carroll, Lewis, *Symbolic Logic, Part I, Elementary*, Second Edition, Macmillan And Co., Ltd., New York: Macmillan & Co., 1896. Bk VIII, Prob. 60, p.124