Tree Trunk Puzzle

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Here is another problem (slightly edited) from the Sherlock Holmes puzzle book by Dr. Watson (aka Tim Dedopulos) ([1] p.106).

Holmes and I were walking along a sleepy lane in Hookland, making our way back to the inn at which we had secured lodgings after scouting out the estates of the supposed major, C. L. Nolan. Up ahead, a team of horses were slowly pulling a chained tree trunk along the lane. Fortunately it had been trimmed of its branches, but it was still an imposing sight.

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When we'd overtaken the thing, Holmes surprised me by turning sharply on his heel and walking back along the trunk. I stopped where I was to watch him. He continued at a steady pace until he'd passed the last of it, then reversed himself once more, and walked back to me.

"Come along, old chap," he said as he walked past. Shaking my head, I duly followed.

"It took me 140 paces to walk from the back of the tree to the front, and just twenty to walk from the front to the back," he declared.

"Well of course," I said. "The tree was moving, after all."

"Precisely," he said. "My pace is one yard in length, so how long is that tree-trunk?"

Can you find the answer?

Solution

We solve this in the usual way by plotting a space-time diagram for the problem (Figure 1), which turns out to be similar to the Marching Cadets and Dog Problem.¹

Let L be the length of the tree log, v_L the speed of the tree advance, and v_S the speed of Sherlock. Further, let T_1 be the time for Sherlock to walk from the back of the tree to its front and T_2 the time for Sherlock to walk from the front of the tree to its back.

Then we have the following relationships from Figure 1

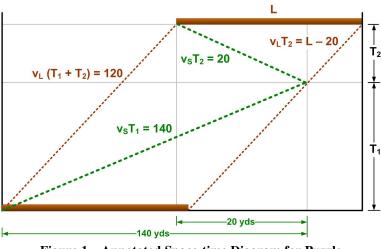


Figure 1 Annotated Space-time Diagram for Puzzle

http://josmfs.net/2019/07/08/marching-cadets-and-dog-problem/

 $v_{s}(T_{1} + T_{2}) = 160$ and $v_{L}(T_{1} + T_{2}) = 120$

which means, eliminating $T_1 + T_2$,

 $3 v_{\rm S} = 4 v_{\rm L}$

Further we have

$$v_{s} T_{2} = 20$$
 and $v_{L} T_{2} = L - 20$

which implies, eliminating T₂,

$$20/v_{\rm S} = (L - 20)/v_{\rm L}$$

or, using $v_L/v_s = 3/4$,

$$L = (3/4)20 + 20 = 35$$
 yards

Dr. Watson's Solution

The answer is 35 yards. We don't know the speed of the team of horses, but the team moves a certain distance, Y, for each pace Holmes takes. So when he has moved 140 yards, the front of the tree has moved 140Y yards. Holmes has walked that distance plus the length of the tree, x, in that time, so in yards, 140 = x + 140Y. In the other direction, Holmes has walked 20 paces, so the tip of the tree has moved 20Y yards. Since they're going in opposite directions, their combined distance equals the length of the tree, and x = 20 + 20Y. So now we have x = 20 + 20Y, and x = 140 - 140Y. So 20 + 20Y = 140 - 140Y, thus 8Y = 6, or Y = 0.75. Since x = 20 + 20Y, then x = 20 + 15 = 35 yards.

The use of the space-time diagram makes solving the problem easier. It allows us to establish variables and relationships among them, which can then be manipulated mechanically using simple symbolic algebra.

References

[1] Dedopulos, Tim, *The Sherlock Holmes Puzzle Collection: The Lost Cases*, Metro Books, Sterling Publishing Co., New York, Carlton Books Ltd., London, 2015.

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