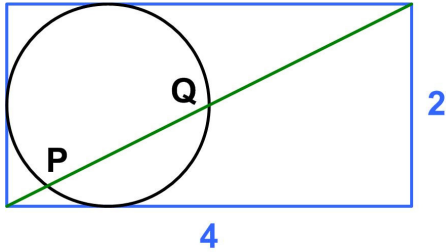


# Circle in Slot Problem

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Here is another UKMT Senior Challenge problem from 2017, which has a straight-forward solution:



The diagram shows a circle of radius 1 touching three sides of a  $2 \times 4$  rectangle. A diagonal of the rectangle intersects the circle at  $P$  and  $Q$ , as shown.

What is the length of the chord  $PQ$ ?

- A  $\sqrt{5}$     B  $4/\sqrt{5}$     C  $\sqrt{5} - 2/\sqrt{5}$     D  $5\sqrt{5}/6$     E 2

## Solution

Figure 1 is annotated to show the relevant relationships. Since the diameter of the circle is 2, the circle reaches halfway across the rectangle, and so the diagonal intersects the circle at its point furthest to the right. Therefore point  $Q$  is one unit above the long side of the rectangle, which means the radius from the center to  $Q$  is parallel to the long side of the rectangle, making the alternating interior angles  $\alpha$  from their intersection with the diagonal equal.

As shown in Figure 2, the red right triangle and smaller blue right triangle are similar. Therefore, the side of the red triangle along  $PQ$  is  $2/\sqrt{5}$ . Since the green right triangle has two sides equal to those in the red right triangle, they are congruent and have their third sides equal. This means the full length of  $PQ$  is  $4/\sqrt{5}$  or answer B.

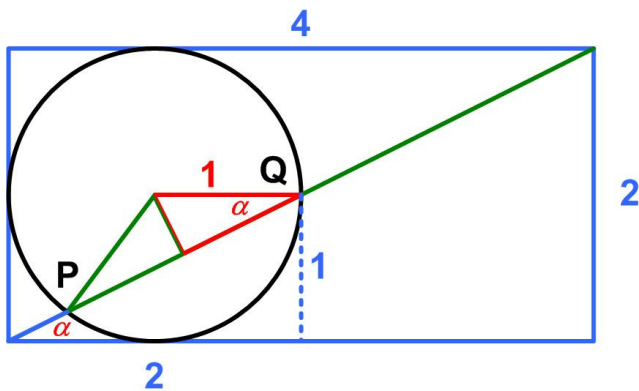


Figure 1 Relevant Relations

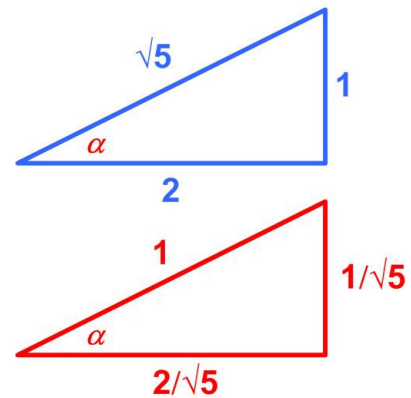


Figure 2 Similar Triangles

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