The *Za'irajah* and Mathematics

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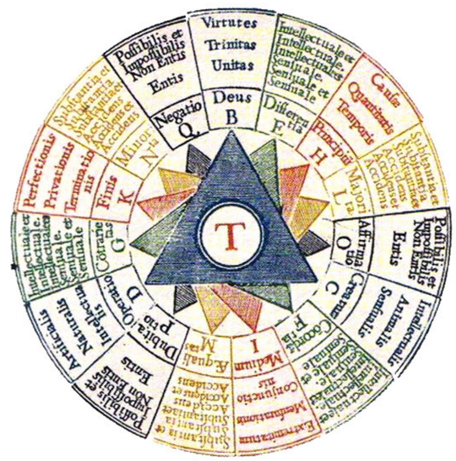
The subtext of this essay might be "word problems," since the stream of thoughts that led to the *za'irajah* (*zairja*) began with a paper I read, while searching for potential problems for this website, on the history of word problems in high school texts in algebra in the 20th and 21st centuries ([2]). The following statement by Lorenat caught my attention ([2] p.169):



Ars Magna, Figura T, Ramon Llull ([1])

> The newer characteristics of how word problems are treated in Long's text [2016] … include adding sympathetic commentary about fear of word problems.
>
>> And of course, there are those dreaded "Word Problems," but I've solved them all for you, so they're painless.
>
> A more extreme example of this is exhibited in the word problem commentary of Michael W. Kelley's *The Idiot's Complete Guide to Algebra*: *Second Edition* from 2007, in which he describes word problems as "a necessary evil of algebra, jammed in there to show you that you can use algebra in 'real life.'" However, Kelley makes no attempt to write "real life" word problems, and criticizes the uselessness of the word problems he does include.

Yes, we all recall that word problems are one of the hardest things about first learning algebra, and yes, the problems are usually contrived. But that is because "real problems" are often too hard to pose and solve at an elementary level, and their messy realistic details get in the way of illustrating the particular algebraic process or capability that is being taught at the moment.

But as far as word problems *per se* are concerned, they should never be disparaged, since they have been the essence of mathematics since its beginning some 5000 years ago. And what is being taught today is not how to solve word problems the way the ancients did, but with the use of the great invention of symbolic algebra in the 16th century.

To gain perspective, which is the best use of the history of math, students should try to solve word problems without using any symbology—just use the words alone. For the most part, that is what Leonardo of Pisa (Fibonacci) did in his *Liber Abaci* (1202) when he introduced the Hindu-Arabic decimal numerals and numerator-denominator style fractions to Europe through a long series of word problems, which in this case were often quite practical. I gave an example in my post "Fibonacci, Chickens, and Proportions".[1] Leonardo employed algebraic methods for solving his problems, but he did not have the symbology yet. Perhaps that delayed the acceptance of his methods for another 200 years, since the merchants continued to prefer using their "hand calculators" (the abacus). It may have been the difficulty of multiplying with an abacus that finally induced acceptance of his procedures. Still, it was another 200 years to about 1600 until symbolic algebra had matured enough to finally dominate mathematical calculations in solving word problems.

Symbolic algebra provided a *mathematical machine* to solve problems, and that brings me to the *za'irajah* and similar devices.

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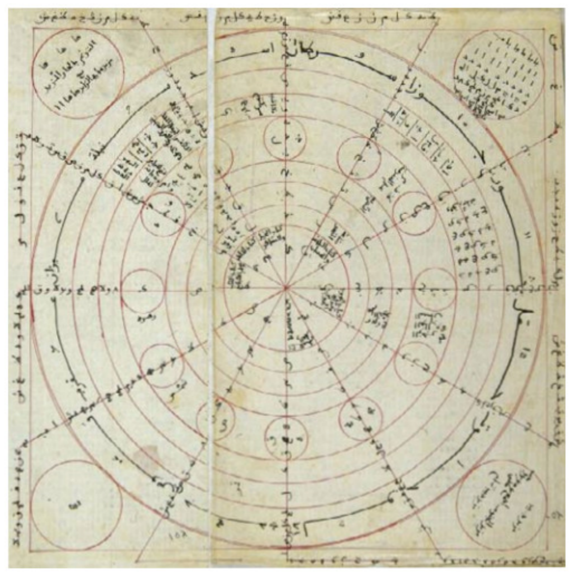
[1] http://josmfs.net/2019/09/06/fibonacci-chickens-and-proportions/

Za'irajah

The urge to have a machine or mechanical method to understand the universe or make predictions has always been with us, if in no other guise than horoscopes. But two particular devices are of note: the *za'irajah* (aka *zairja*) and Lull's *Ars Magna* (*Great Art*). The *za'irajah* is discussed in Ibn Khaldun's *Muqaddimah* (1377). Its origin is attributed to Abu1-'Abbas as-Sabti, who Khaldun claimed lived at the end of the 12th century. It has been suggested ([3] p.216) that the 13th century Spanish mystic Ramon Lull used the idea of the *za'irajah* to construct his *logical machine* of concentric wheels, the *Ars Magna* (1274) ([4]), which he employed as an objective device to try to convert the Muslims of North Africa to Christianity (of course, to no avail). (See an example of one of Lull's wheels in the T Figure at the beginning of this essay.)

It is Ibn Khaldun's comparison of the *za'irajah* with mathematics that is of interest here. The following is an excerpt from Rosenthal's 1958 translation ([5] Vol I. Section 6.):

> Another technical rule for alleged discovery of the supernatural is the *za'irajah* which is called *"Za'irajah* of the world." …



> The *za'irajah* is a remarkable technical procedure. Many distinguished people have shown great interest in using it for supernatural information, with the help of the well-known enigmatic operation that goes with it. … The form of the *za'irajah* they use is a large circle that encloses other concentric circles for the spheres, the elements, the created things, the *spiritualia,* as well as other types of beings and sciences. … The *za'irajah* is surrounded by verses …. They describe the procedure which must be followed to discover the answer to a particular inquiry from the *za'irajah.* However, since the verses express their meaning in riddles, they lack clarity. …

> We have seen many distinguished people jump at (the opportunity for) supernatural discoveries through (the *za'irajah)* …. They think that correspondence (in form) between question and answer shows correspondence in actuality. This is not correct, because, as was mentioned before, perception of the supernatural cannot be attained by means of any technique whatever. … Intelligent persons may have discovered the relationships among these things, and, as a result, have obtained information about the unknown through them. Finding out relationships between things is the secret (means) whereby the soul obtains knowledge of the unknown from the known. It is a way to obtain such knowledge, especially suited to people of (mystical) training. This (training) gives the intellect added power for analogical reasoning and thinking, as has been explained before several times. It is in this sense that *za'irajahs* are usually ascribed to people of (mystical) training. …

> Many people lack the understanding necessary for belief in the genuineness of the operation and its effectiveness in discovering the object of inquiry. They deny its soundness and believe that it is hocus-pocus. …

> Many an operation with numbers, which are the clearest things in the world, is difficult to grasp, because the (existing) relations are difficult to establish and intricate. This is the case to a much greater degree here, where the relations are so intricate and strange.

Let us mention a problem that will to some degree illustrate the point just stated.

Take a number of dirhams and place beside each dirham three fals. Then, take all the fals and buy a fowl with them. Then, buy fowls with all the dirhams for the same price that the first bird cost. How many fowls will you have bought?

The answer is nine. As you know, a dirham has twenty-four fals, three fals are one-eighth of a dirham, one is eight times one-eighth. Adding up one-eighth of each dirham buys one fowl. This means eight fowls (for the dirhams), as one is eight times one-eighth. Add another fowl, the one that was bought originally for the additional fals and that determined the price of the fowls bought with the dirhams. This makes nine. It is clear how the unknown answer was implied in the relations that existed between the numerical data indicated in the problem. This and similar (things) are at first suspected as belonging to the realm of the supernatural, which cannot be known.

It is thus obvious that it is from the relations existing among the data that one finds out the unknown from the known. This, however, applies only to events occurring in (the world of) existence or in science. Things of the future belong to the supernatural and cannot be known unless the causes for their happening are known and we have trustworthy information about it.

If this is clear, it follows that all the operations of the *za'irajah* serve merely to discover the words of the answer in the words of the question. As we have seen, it is a question of producing from a given arrangement of letters another arrangement of letters. The secret here lies in the existence of a relationship between the two (different arrangements of letters). Someone may be aware of it, whereas someone else may not be aware of it. Those who know the existing relationship can easily discover the answer with the help of the stated rules.

Another Word Problem Example

There are several things of interest in the excerpt. First, consider the word problem included in the text:

> Take a number of dirhams and place beside each dirham three fals. Then, take all the fals and buy a fowl with them. Then, buy fowls with all the dirhams for the same price that the first bird cost. How many fowls will you have bought?

This appears to be a typical word problem with its somewhat obscure description. Now consider the solution given in the text solely without symbols:

> The answer is nine. As you know, a dirham has twenty-four fals, three fals are one-eighth of a dirham, one is eight times one-eighth. Adding up one-eighth of each dirham buys one fowl. This means eight fowls (for the dirhams), as one is eight times one-eighth. Add another fowl, the one that was bought originally for the additional fals and that determined the price of the fowls bought with the dirhams. This makes nine.

How clear is that? Here is the somewhat literal translation into mathematical symbols provided by the translator, Rosenthal:

> In modem symbols, x being the number of fowls, y the number of dirhams:

$$y \bullet 1/8 = 1$$

$$y + y \bullet 1/8 = x$$

$$x = 8 + 1.$$

I still find this a bit obscure and propose the following alternative:

24 fals = 1 dirham. N = number of dirhams. Using 3 fals for each of N dirhams to buy 1 fowl means it costs $3N$ fals per fowl. We have N dirhams (= $24N$ fals). Therefore the number of fowls bought with the N dirhams is $24N$ fals / ($3N$ fals/fowl) = 8 fowls. So together with the first purchased fowl, that is 9 fowls altogether.

In any case, it is evident that translating the word problem into symbolic algebra greatly clarifies the solution.

Logical/Mathematical Machines

The second thing to notice in the description of the *za'irajah* is the highlighted text that indicates the answer to the problem is inherent in the statement of the problem, and the process of revealing the solution appears to the untutored eye to be like magic or the supernatural. The *za'irajah* and Lull's *Ars Magna* are logic machines, in that they answer verbal questions by applying the rules of logic in a mechanical way. The operations of symbolic algebra provide a mathematical machine. Once the problem has been translated into arithmetic statements, "blindly" applying the rules of arithmetic yields the solution, just like a machine.

The early symbolic algebra mathematical machine of 1600 transformed almost 5000 years of doing mathematical problems. Its results were so powerful, that within another 100 years it had been expanded to solve geometric problems (analytic geometry) and physical problems involving change and the infinite (calculus). Today it is a monster machine that is applied to virtually every aspect of our existence. Its predictions put satellites in space and model the invisible patterns of electricity and magnetism that drive our smartphones. I had discussed before how the essence of mathematics was mathematical representations or models.[2] That was just another way of describing the vast mathematical machine we have developed over the last 400 years.

So never belittle the notorious word problem. Some mathematicians suggest teaching the algebraic methods of word problem solutions is archaic and obsolete, since now we have software that handles the symbolic operations, even for the methods of differentiation in calculus ([6]). Clearly, I see the continued importance of these methods. And the plethora of problems on this website attests to the challenge and satisfaction that still adhere to solving word problems that span the millennia.

Digression (Rant). If the power of math so evidently comes from its symbols (models) and if this discovery and development is the essence of math, why on earth does virtually every book written about math for the layman extol the absence of any equation marring its pages? Half the time when I try to read such books, I can barely understand what they are saying. It is like I am thrown back 500 years to pre-symbol times. I can't believe anyone graduating from today's public schools can find pre-Renaissance mathematical word descriptions easier to understand than simple equations. (Sorry, I had to get that off my chest.)

References

[1] Ramón Lull, *Cuatro Obras*: *El desconsuelo; Canto de Ramón; Del nacimiento de Jesús Niño; El extravagante*, Edición de Julia Butiñá, Centro de Lingüística Aplicada Atenea, November 2013.

[2] Lorenat, Jemma et al., "From Carriage Wheels to Interest Rates: The Evolution of Word Problems in Algebra Textbooks from 1901 to Today," *Journal of Humanistic Mathematics*, Volume 10, Issue 1, January 2020, pp. 145-180. (https://scholarship.claremont.edu/jhm/vol10/iss1/8)

[2] My post on the "Essence of Mathematics" (http://josmfs.net/2019/03/03/the-essence-of-mathematics/)

[3] Link, David (2010). “Scrambling T-R-U-T-H: Rotating Letters as a Material Form of Thought”, in: *Variantology 4. On Deep Time Relations of Arts, Sciences and Technologies in the Arabic–Islamic World*, eds. Siegfried Zielinski and Eckhard Fürlus (Cologne: König, 2010): 215–266 (http://www.alpha60.de/research/scrambling_truth/DavidLink_ScramblingTruth2010_100dpi.pdf, retrieved 2/16/2020)

[4] Gardner, Martin, “The Ars Magna of Ramon Lull,” *Logic Machines and Diagrams,* McGraw-Hill, 1958. pp.1-27. (https://monoskop.org/images/e/e6/Gardner_Martin_Logic_Machines_and_Diagrams.pdf)

[5] Ibn Khaldun, Franz Rosenthal tr., *The Muqaddimah: Introduction to History*, Bollingen Foundation Inc., New York, N. Y., 1958. (https://asadullahali.files.wordpress.com/2012/10/ibn_khaldun-al_muqaddimah.pdf)

[6] Devlin, Keith, “All The Mathematical Methods I Learned In My University Math Degree Became Obsolete In My Lifetime”, *Huffington Post*, 23 January 2017 (https://www.huffingtonpost.com/entry/all-the-mathematical-methods-i-learned-in-my-university_us_58693ef9e4b014e7c72ee248?ncid=engmodushpmg00000004). To be fair, Devlin says:

> The shift began with the introduction of the electronic calculator in the 1960s, which rendered obsolete the need for humans to master the ancient art of mental arithmetical calculation. Over the succeeding decades, the scope of algorithms developed to perform mathematical procedures steadily expanded, culminating in the creation of desktop packages such as *Mathematica* and cloud-based systems such as *Wolfram Alpha* that can execute pretty well any mathematical procedure, solving—accurately and in a fraction of a second—any mathematical problem formulated with sufficient precision (a bar that allows in all the exam questions I and any other math student faced throughout our entire school and university careers).
>
> So what, then, remains in mathematics that people need to master? The answer is, the set of skills required to make effective use of those powerful new (procedural) mathematical tools we can access from our smartphone. …

But it is not clear to me from the essay how or what skills should be developed. I would hope solving word problems would still be found to be necessary, and to my mind that still requires understanding the arithmetic or other mathematical steps needed to accomplish this.