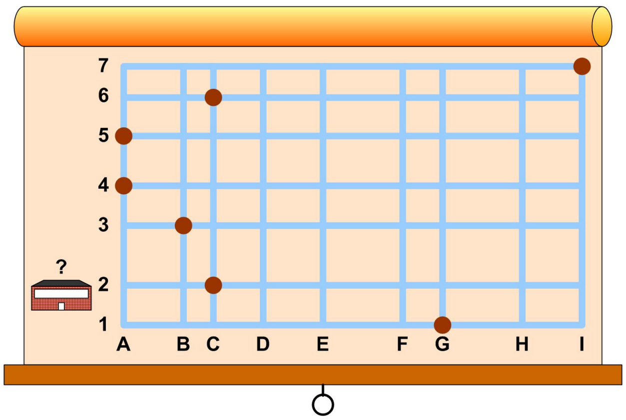
Factory Location Problem

10 October 2019

Jim Stevenson



This is a somewhat elegant problem from the 1987 *Discover* magazine's Brain Bogglers by Michael Stueben ([1]):

> Each dot in the figure at left represents a factory. On which of the city's 63 intersections should a warehouse be built to make the total distance between it and all the factors as short as possible? (A much simpler solution than counting and totaling the distances is available.)

Note that the distance is the taxicab distance I discussed in my article "South Dakota Travel Problem"[1] rather than the distance along straight lines between the warehouse and factories.

My Solution

Start at the factory at I Street and 7th Avenue in the upper right-hand corner of the grid. Now move along 7th Ave. one block west to H Street. The distance to 6 of the factories will be diminished by 1 block and the distance to one factory (at 7th and I) will be increased, thus given a net decrease to the total sum of distances of 5. We continue in this way along 7th Avenue until we reach an intersection where any move left or right increases the sum of distances. Now we progress down the avenues until we reach another intersection where any move up or down increases the sum of distances. We have reached the intersection of minimum sum of distances to all the factories.

The following table gives the solution

Moves	Change in Each Factory Distance	Net Change	Cumulative Change in Total Distance	New Warehouse Location
Initial position	0	0	0	I St 7th Ave
1 block west	- 6 blocks + 1 block	- 5 blocks	- 5 blocks	H St 7th Ave
1 block west	- 6 blocks + 1 block	- 5 blocks	- 10 blocks	G St 7th Ave
1 block west	- 5 blocks + 2 blocks	- 3 blocks	- 13 blocks	F St 7th Ave
1 block west	- 5 blocks + 2 blocks	- 3 blocks	- 16 blocks	E St 7th Ave
1 block west	- 5 blocks + 2 blocks	- 3 blocks	- 19 blocks	D St 7th Ave
1 block west	- 5 blocks + 2 blocks	- 3 blocks	- 22 blocks	**C St** 7th Ave
(1 block west	- 3 blocks + 4 blocks	+1 blocks	- 21 blocks	B St 7th Ave)
1 block south	- 6 blocks + 1 block	- 5 blocks	- 27 blocks	**C St** 6th Ave
1 block south	- 5 blocks + 2 blocks	- 3 blocks	- 30 blocks	**C St** 5th Ave

[1] http://josmfs.net/2018/12/28/south-dakota-travel-problem/

Moves	Change in Each Factory Distance	Net Change	Cumulative Change in Total Distance	New Warehouse Location
1 block south	- 4 blocks + 3 blocks	- 1 block	- 31 blocks	**C St 4th Ave**
(1 block south	- 3 blocks + 4 blocks	+ 1 block	- 30 blocks	**C St** 3th Ave)

So we arrive at C Street and 4th Avenue as the location for the warehouse that has the minimum sum of distances to all the factories. Any move, east or west, north or south, from this intersection will increase the total sum of distances.

Comment. The approach shown here is an example of the direct search method in optimization where the minimum or maximum of a discrete or non-differentiable function of several variables is sought. One starts at a point and considers the values of the function at a discrete number of neighboring points. One then moves to the point that gives the least (greatest) value of the function and repeats the process until no step produces a smaller (larger) value of the function.

Brain Bogglers Solution

My solution is basically equivalent to that of Brain Bogglers.

C-4. This problem is best solved by the democratic process. Lay a pencil vertically on A Street. Ask yourself whether, if you were the owner of each factory, you would want the warehouse to be located east of the pencil. You'd vote yes for five factories, no for two. Majority rules. At B Street the vote would be four in favor, three opposed. But at C Street only two of the seven votes would be in favor of putting the warehouse east of the pencil. So the warehouse should go on C Street. Now repeat the process for the horizontal avenues. Again, under the power of majority rule, the pencil would stop at 4th Avenue.

References

[1] Stueben, Michael, "Brain Bogglers," *Discover*, March 1987