Trains – Pickleminster to Quickville

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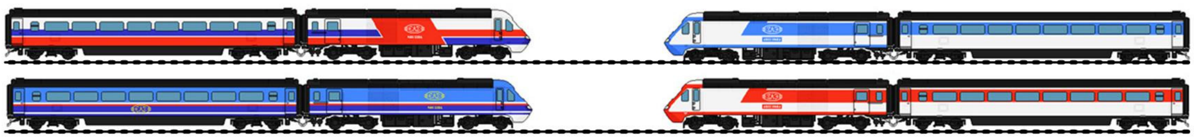
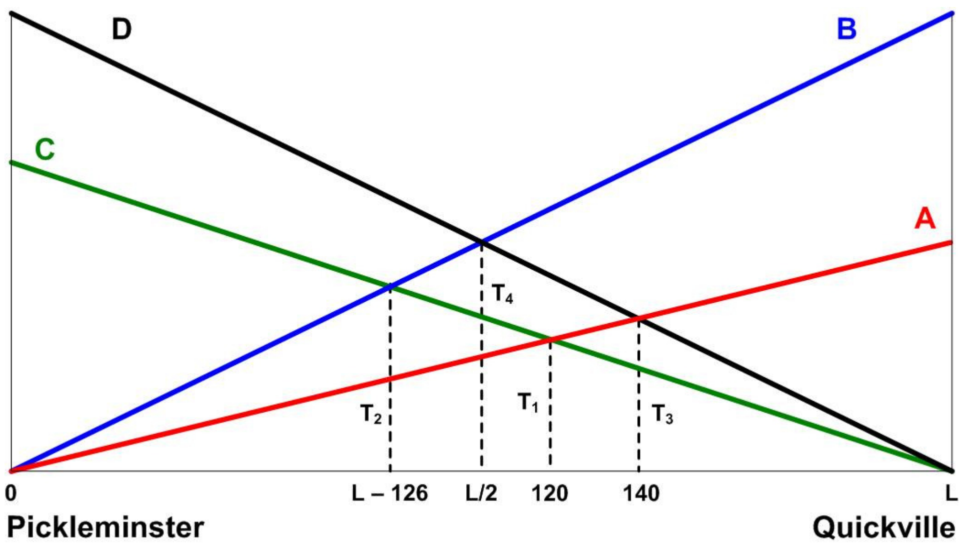


This is another train puzzle by H. E. Dudeney. This one has some hairy arithmetic. ([1] p.20)

68. PICKLEMINSTER TO QUICKVILLE

Two trains, A and B, leave Pickleminster for Quickville at the same time as two trains, C and D, leave Quickville for Pickleminster. A passes C 120 miles from Pickleminster and D 140 miles from Pickleminster. B passes C 126 miles from Quickville and D half way between Pickleminster and Quickville. Now, what is the distance from Pickleminster to Quickville? Every train runs uniformly at an ordinary rate.

Solution



The figure represents the setup for the problem. We can derive a number of proportions from similar triangles:

$$\frac{T_2}{L-126} = \frac{T_4}{L/2} \quad \text{and} \quad \frac{T_3}{L-140} = \frac{T_4}{L/2} \tag{1}$$

$$\frac{T_1}{120} = \frac{T_3}{140} \quad \text{and} \quad \frac{T_2}{126} = \frac{T_1}{L-120} \tag{2}$$

(1) and (2) imply, respectively,

$$\frac{T_2}{L-126} = \frac{T_3}{L-140} \quad \text{and} \quad \frac{(L-120)\,T_2}{126} = \frac{120\,T_3}{140} \tag{3}$$

and thus

$$\frac{T_3}{T_2} = \frac{L - 140}{L - 126} \qquad \text{and} \qquad \frac{T_3}{T_2} = \frac{7\,(L - 120)}{6 \cdot 126} \tag{4}$$

Therefore, $$6 \cdot 126\,(L - 140) = 7\,(L - 120)(\,L - 126)$$

or $$L^2 - (6 \cdot 20 + 6 \cdot 21)D + 6 \cdot 20 \cdot 6 \cdot 21 = 6 \cdot 18\,L - 6 \cdot 18 \cdot 140$$

or $$L^{2} - 6(20 + 21 + 18)\,L + 6 \cdot 20 \cdot 6 \cdot 21 + 6 \cdot 20 \cdot 6 \cdot 21 = 0$$

or $$L^{2} - 6 \cdot 59\,L + 2 \cdot 6^2 \cdot 20 \cdot 21 = 0.$$

We apply the quadratic formula for the solution to the equation of the form $a\,x^2 + b\,x + c = 0$ where $b = -\,6 \cdot 59$ and $b^2 - 4ac = (6 \cdot 59)^2 - 4 \cdot (2 \cdot 6^2 \cdot 20 \cdot 21) = 6^2 \cdot (59^2 - 4 \cdot 2 \cdot 20 \cdot 21) = 6^2 \cdot 11^2$. So

$$L = ½\,(6 \cdot 59 \pm 6 \cdot 11) = 210 \text{ or } 144 \text{ miles}$$

As Dudeney argues, if 144 miles were the answer, then if A traveled 140 miles in an hour, then B and D would only travel 4 miles in an hour. These are not “ordinary rates” as Dudeney asserts. So 144 miles is eliminated as an answer, leaving the distance L = 210 miles.

References

[1] Dudeney, Henry Ernest, *536 Puzzles & Curious Problems*, Edited By Martin Gardner, Charles Scribner's Sons, New York, 1967.