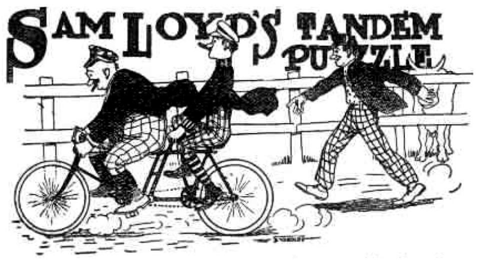
Tandem Bicycle Puzzle

3 July 2019

Jim Stevenson



A glutton for punishment I considered another Sam Loyd puzzle ([1] p.322):

> Three men had a tandem and wished to go just forty miles. It could complete the journey with two passengers in one hour, but could not carry the three persons at one time. Well, one who was a good pedestrian, could walk at the rate of a mile in ten minutes; another could walk in fifteen minutes, and the other in twenty. What would be the best possible time in which all three could get to the end of their journey?

My Solution

Figure 1 shows a space-time diagram for the three walkers and the bicycle. We will designate their speeds as v_S, v_M, v_Q, and v_B for the slow, medium, and quick walkers, and the bicycle, respectively.

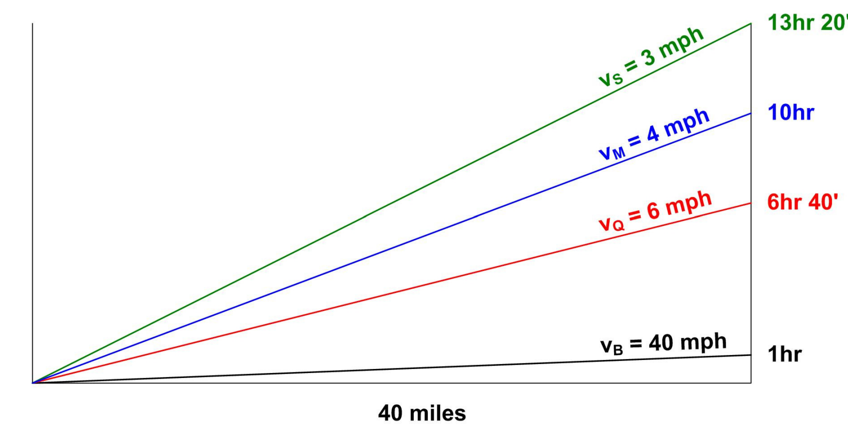


Figure 1 Problem Statement Space-Time Diagram

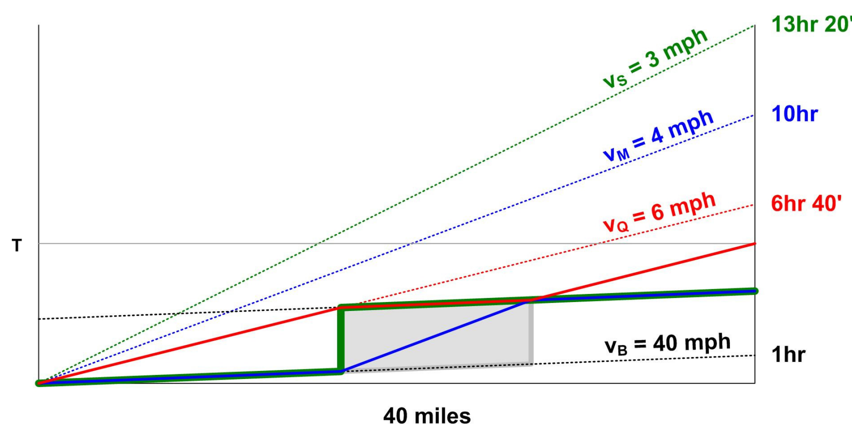


Figure 2 Figure 1 Augmented with Example of Bicycle Use

Figure 2 shows an example of a possible solution to the problem. *We have assumed that the slowest walker remains on the tandem bicycle throughout.* This example shows the quick walker starting out walking, and both the slow and medium walkers using the bicycle up to a certain point, where the medium walker gets off and starts walking. If the slow walker continues to ride the bicycle, then he will arrive at the end of 40 miles in one hour and none of the other walkers will ever catch up and be able to use the bicycle. So, in order for the quick walker to be able to share the bicycle, the slow walker must wait until the quick walker catches up at the point where the medium walker got off the bicycle. Then they both ride to the point where they meet the medium walker again. The quick walker gets off and the medium walker gets back on the bicycle and rides with the slow walker to the end, where they wait for the quick walker to finally arrive at the total time T.

Note that our assumption that the slow walker always uses the tandem bicycle essentially reduces the problem to two riders with a single-seat bicycle, where when one rider is done using it, the bicycle must remain where that rider left it until the other rider catches up to it.

Consider the grey parallelogram in Figure 2. A more detailed version is shown in Figure 3. This diagram shows what has to happen if a bicycle rider gets off and walks to the next rendezvous with the bicycle. The vertical distance is the wait time for the bicycle being left for the next rider. Figure 4 shows the proportional space-time plots for each walker.

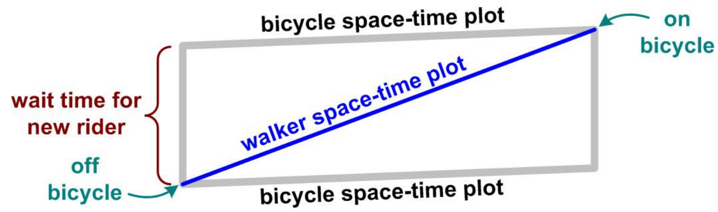


Figure 3 Space-time Plot for Trade-off Between Walking and Bicycle

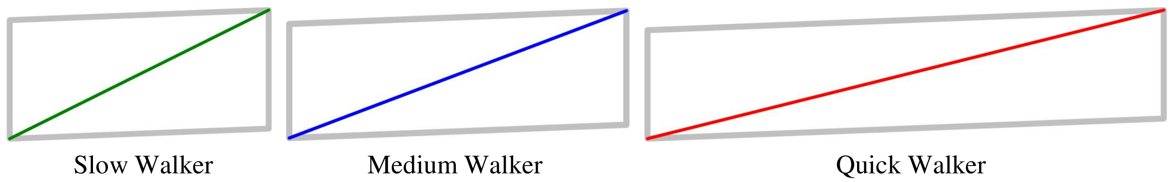


Figure 4 Walk-Bicycle Space-time Diagrams for the Three Walkers

Figure 5 shows what happens if the medium walker abandons the bicycle later. The quick walker will have walked further and longer, thus causing the wait time for the bicycle to grow longer. But the quick walker gets to abandon the bicycle later and so arrive earlier than before. The medium walker still arrives on the bicycle, but at a later time. However, that time is still less than the quick walker's arrival time. So the quick walker's arrival time is still the time that all three walkers have arrived at their destination.

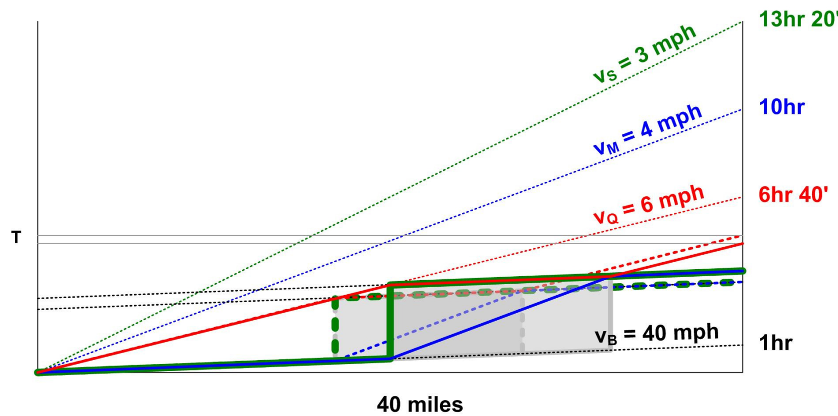


Figure 5 Medium Walker Abandons Bicycle Later

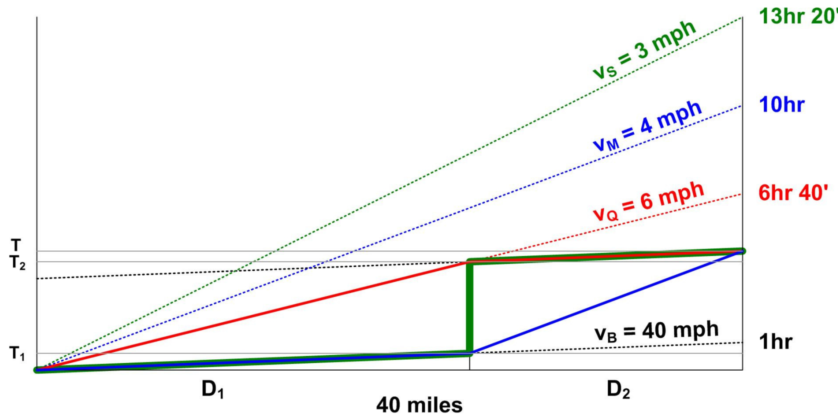


Figure 6 Final Position—Solution

In terms of the space-time parallelogram (Figure 3), it expands as it gets closer to the right edge of the plot. At which point the quick walker has ridden the bicycle all the way to the end, that is, the red walking line in the diagram coalesces with the bicycle line (disappears) at the minimum value for T (Figure 6).

In other words it seems reasonable that the optimal result would be to have the bicyclists and the last walker arrive at the destination at the same time. That would mean the overall time would be the time for the bicycle to make the journey (1 hour) plus the time the bicycle was waiting to be ridden ($T_2 - T_1$).

There is an alternative solution that results in the same final time T. Namely, the quick walker rides the bicycle first and then walks, whereas the medium walker walks first and then rides. The distances D_1 and D_2 and corresponding times T_1 and T_2 are different, but $D_1 + D_2$ still equals 40 miles and $T_2 - T_1$ is the same as before. Just swap their respective walk-bicycle space-time diagrams (parallelograms). That is, the distance D_1 in Figure 7 equals the distance D_2 in Figure 6, and so the distance D_2 in Figure 7 equals the distance D_1 in Figure 6.

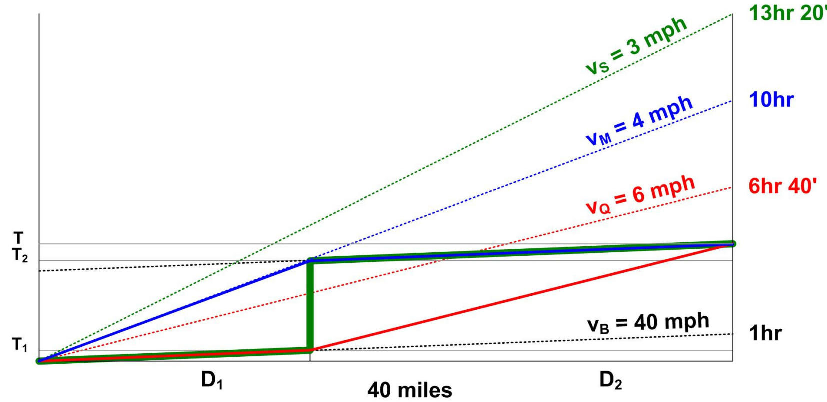


Figure 7 Alternative Solution with the Same Outcome

I confess to a little chagrin that I could not come up with an analytic proof for the minimal time T, but I feel this graphical method is sufficiently robust to solve the problem. (I also, just for completion, considered the walk-bicycle space-time diagram for the slow walker, but all positionings resulted in larger values for the final time T, as expected.) Now for the computations arriving at the answer for the time T.

Computations

I will use the values labeled in Figure 6. Then in hours

$$T = T_2 - T_1 + 1 \quad (1)$$

Then for the distances we have

$$D_1 = v_B T_1, \quad D_1 = v_Q T_2 \quad (2)$$

$$D_2 = v_M (T - T_1) = v_M (T_2 - 2T_1 + 1) \quad (3)$$

$$40 = D_1 + D_2 \quad (4)$$

From equations (2) – (4) we have

$$\frac{40}{v_M} = D_1\left(\frac{1}{v_M} + \frac{1}{v_Q} - \frac{2}{v_B}\right) + 1$$

or $$9 = D_1 \, 11/30$$

Thus $$D_1 = 24.55 \text{ mi} \quad (5)$$

Therefore $$T_2 - T_1 = 45/11 - 27/44 = 153/44$$

and $$T = T_2 - T_1 + 1 = 197/44 = 4.48 \text{ hr} = 4 \text{ hr } 29'$$

Sam Loyd's Solution

Okay, so how did Sam Loyd solve the problem? ([1] p.382)

ANSWER TO TANDEM PUZZLE. Herman[1] contributed three-quarters of the speed for the first third, or one-quarter of the motive power[2] required for the necessary energy[3] for the next quarter, or one-twelfth of the total. Thus for the seven-twelfths of the journey he gave four-twelfths of the energy required for the entire trip. In going the remaining five-twelfths of the way Herman must supply two-fifths of the power or one-ninth of the whole, which, together with his contributions of one-quarter and one-twelfth, make up one-half.

This strikes me as complete gibberish. I checked all the surrounding answers and could not find any other that might have been the actual answer, nor the question this might be the answer to. And besides the discussion of factors that were never an issue, there seems to be no actual answer provided as to the shortest time T. Again, this answer captures the issues I have with Sam Loyd: extra factors not mentioned in the original problem statement, complete English language solution without equations, and not only no "solution" but also no "answer" even. (It really looks more and more like this is an answer to another problem. But I scanned the entire volume for a reference to "Herman", and there wasn't one other than in the answer to the Tandem Puzzle, nor was there a puzzle that seemed to fit this solution.)

Martin Gardner's Problem Statement

I found the Tandem Bicycle Problem in the book of Sam Loyd puzzles Martin Gardner edited ([2] p.88):

123 Tandem Bicycle

Three men wish to go forty miles on a tandem bicycle that will carry no more than two at a time while the third man is walking. One man, call him A, walks at a rate of one mile in ten minutes, B can walk a mile in fifteen minutes, and C can walk a mile in twenty minutes. The bicycle travels at forty miles an hour regardless of which pair is riding it. What is the shortest time for all three men to make the trip, assuming, of course, that they use the most efficient method of combining walking and cycling?

Notice the Gardner makes a point that "The bicycle travels at forty miles an hour *regardless of which pair is riding it*." So all Sam Loyd's discussion about the various motive powers and energy of the walkers is irrelevant. Unfortunately, I do not have a copy of Gardner's book, so I don't know how he solved the problem or whether he agrees with my answer. Maybe I can corroborate it in the future.

References

[1] Loyd, Sam, *Cyclopedia of Puzzles*, Lamb Publishing, New York, 1914. p.322 (http://djm.cc/library/Cyclopedia_of_Puzzles_Loyd.pdf)

[2] Gardner, Martin, ed., *More Mathematical Puzzles of Sam Loyd*, Vol. 2, Dover Publications, New York, 1960.

[1] JOS: Who's Herman?

[2] JOS: Since when does "motive power" enter the picture? How does that change the speeds which I assumed were givens?

[3] JOS: Now we care about energy?