

Logging Problem

4 July 2019

Jim Stevenson

This is a delightful little problem from Dick Hess ([1] p.26) that exercises one's basic facility with logarithms.

Define x as $x = \log_{16}7 \times \log_{49}625$. Find an expression for $\log_{10}2$ in terms of x where the only constants appearing are integers.

Solution

Let $u = \log_{16}7$ and $v = \log_{49}625$, so $x = uv$.

Then $16^u = 7$ or $2^{4u} = 7$.

And $49^v = 625$ or $7^{2v} = 25^2 = 5^4$ or $7^v = 5^2$

Therefore $(2^{4u})^v = 2^{4x} = 5^2$

and $2^{2x} = 5 = 10/2$

So $2^{2x+1} = 10$

or $\log_{10}2 = 1 / (2x + 1)$



newvitruvian.com

References

- [1] Hess, Dick, *The Population Explosion and Other Mathematical Puzzles*, World Scientific Pub., 2016