Ladies' Diary Problem

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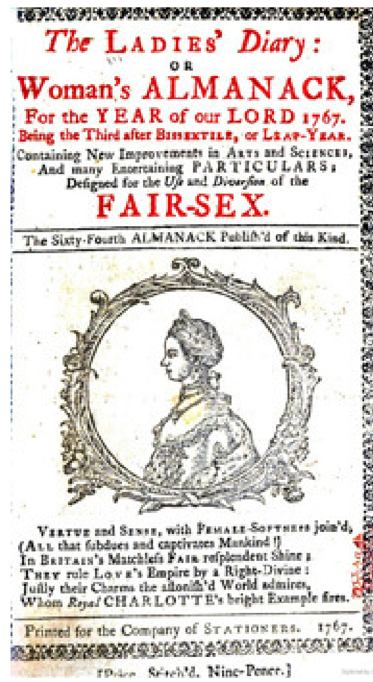
An amazing publication was conceived primarily for women at the beginning of the 18th century in 1704 and was called *The Ladies' Diary or Woman's Almanack*. The publication survived for almost 150 years to 1841, through a number of editors beginning with John Tipper. As Frank Swetz's *Convergence* article explained ([2]):



> One such enterprising editor was John Tipper (1663-1713), a Coventry schoolmaster and sometime textbook author. Mindful of the growing leisure class of young women and their intellectual restlessness, he conceived of an almanac just for women, the *Ladies' Diary or Woman's Almanack.* At this time there were no specific periodicals or magazines for women. In the latter part of the seventeenth century, a publisher, John Dutton, had attempted two journals designed for women, but his content demeaned and preached to its female readers. These journals failed quickly. Tipper believed he could cater successfully to the "Fair Sex" by supplying "genteel" subject matter, such as household tips, recipes, health advice, poems, and "delightful" romantic stories, along with the standard calendar and chronological reckonings of an almanac. … One prominent feature of Tipper's *Ladies Diary* was the inclusion of "enigmas," or word puzzles, a pastime that was all the rage among English women at the time. …
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> By 1709, the almanac's readership was requesting more of these word puzzles, both in enigmatic form and arithmetical in nature. Tipper complied, noting a change in policy and set new standards that would radically alter the mathematical and educational impact of his publication. First, he revised his content to comply with his female readers' preference for intellectually challenging problem-solving situations over cooking advice. Secondly, he cautioned that the arithmetical problems be within the capacity of his women readers.
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> The reserved and socially constrained "Ladies" of England wanted to do mathematics! John Tipper's accommodation and solicitation for mathematics problems acknowledged the lack of mathematical training provided young women. If they had acquired skills in mathematics, it was usually through self-study and or tutoring by liberal-minded parents or a governess. The prevailing conception held was that females could not engage in anything scientific, let alone mathematics; exposure to it would "fever their brains." However, the first prize problem was sent in by a man and answered correctly by Mrs. Mary Wright. Over several years, Mrs. Wright would distinguish herself by solving many of the mathematical problems set in the *Diary.*

Here was the Prize problem and Mrs. Wright's solution ([1] p.12):

VI. Prize Question 16. (proposed in 1710, answered in 1711)

Walking through Cheapside, London, on the first day of May, 1709, the sun shining brightly, I was desirous to know the height of Bow Steeple. I accordingly measured its shadow just as the clock was striking twelve, and found its length to be $253^1/_8$ feet; it is required from thence to find the steeple's height.

***Answered by Mrs.* Mary Wright**

May 1, 1709

	°	′	″
Sun's longitude, from its ingress into aries	51	28	0
Oblique angle of the ecliptic and equator	23	29	0
Thence the declination that day	18	9	45
Consequently its merid. altitude in lat. 51° 32′	56	37	45
The complement thereof to 90 is	33	22	15

Then as the sine of the angle 33° 22′ 15″.
To the base 253.125 feet.
So is the sine of the angle 56° 37′ 45″.
To the perpendicular 384.307 feet the height of the steeple.

Note. The true height of Bow steeple is 225 feet, for which at first I had proportioned the length of the shadow, but upon second thoughts I altered it, for fear some, who had read its height in history, should claim the reward, without having art enough to investigate it by trigonometry.[1]

The solution shows a formidable command of astronomy and trigonometry, as well as the ability to navigate quite complicated computations (all without calculators!). I noticed that many problems in the *Diary* entailed spherical geometry for ocean navigation and knowledge of the terminology and meaning of astronomical situations. Geometry problems often involved cones or frustrums of cones, my least favorite geometric objects. Throughout, the problems illustrated the British love of hairy calculations with little effort to have clean answers.

Here is the main object of this essay: a nice problem posed by Mrs. Mary Nelson, nee Mary Wright.[2] What makes this problem nice is that it does have a clean answer.

VIII. Question 72 ***by Mrs. Mary Nelson*** (proposed in 1719, answered in 1720)

A prize was divided by a captain among his crew in the following manner: the first took £1 and one hundredth part of the remainder; the second £2 and one hundredth part of the remainder; the third £3 and one hundredth part of the remainder; and they proceeded in this manner to the last, who took all that was left, and it was then found that the prize had by this means been equally divided amongst the crew. Now if the number of men of which the crew consisted be added to the number of pounds in each share, the square of that sum will be four times the number of pounds in the chest: How many men did the crew consist of, and what was each share?

Answer.

The men were 99, and had £99 each. The whole £9801.

Solution.

[provided by Dr. Charles Hutton, Emeritus Professor of the Mathematics in the Royal Military Academy at Woolwich]

Put x for the number of pounds in the prize: Then, by the question

$$1+\frac{x-1}{100}=\frac{99+x}{100}$$

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[1] JOS: Is this note written by Mrs. Wright or the proposer of the problem? Or are they one and the same? Swetz's essay suggests they are different with the problem being posed by a man.

[2] JOS: ([3] p.17) "Costa convincingly argues that Mrs. Mary Nelson [Leybourn, 1817, I, 24] was probably the married name of Mary Wright and Anna Philomathes [Diary, 1719] was probably her sister Anna Wright [Diary, 1711, 31; Costa, 2000, 210]"

equals the first man's share, and consequently the sum afterwards left equals

$$x - \frac{99 + x}{100} = \frac{99x - 99}{100};$$

again, the second man's share will be

$$2 + \frac{\frac{99x - 99}{100} - 2}{100} = \frac{198 + \frac{99x - 99}{100}}{100} = \frac{19701 + 99x}{10000},$$

which must be equal to that of the former, that is,

$$\frac{99 + x}{100} = \frac{19701 + 99x}{10000};$$

hence

$$9900 + 100x = 19701 + 99x,$$

and

$$x = 19701 - 9900 = 9801$$

equals the number of pounds in the prize. And

$$\frac{99 + x}{100} = 99$$

equals each man's share.

Again, putting z equal to the number of men, by the question it will be

$$(z + 99)^2 = 4 \times 9801;$$

hence, by extracting the root, $z + 99 = 2 \times 99$, and $z = 99$ equals the number of men the same with the number of pounds in each man's share.

[Leybourn, 1817, I, pp.94–95; Algebra]

My Solution

I had originally seen the problem in Albree's article ([3]) where it was presented without a solution. So I solved it before seeing Hutton's solution in Leybourn's compilation. The detailed narration of the division of the prize money between each crew member was reminiscent of other such problems, and so I began a careful record of each step—ultimately arriving at a recursive formulation as follows. Let P be the amount of the prize and n the number of men in the crew. Then

Crewman	Share	Remainder
First	$1 + (P - 1)/100$	$P_1 = P - 1 - (P - 1)/100 = (1 - 1/100)(P - 1)$
Second	$2 + (P_1 - 2)/100$	$P_2 = P_1 - 2 - (P_1 - 2)/100 = (1 - 1/100)(P_1 - 2)$
…		
k^{th}	$k + (P_{k-1} - k)/100$	$P_k = (1 - 1/100)(P_{k-1} - k)$

This was going to be complicated, but then I realized the implications of the phrase "the prize had by this means been equally divided amongst the crew" and the following phrase "if the number of men of which the crew consisted be added to the number of pounds in each share, the square of that sum will be four times the number of pounds in the chest". These phrases meant I could ignore the recursive relation for a while and do some direct calculations. That is, each crewman's equal share was P/n and so the second phrase translated into

$$(P/n + n)^2 = 4\,P$$

$$(P + n^2)^2 = 4\,P\,n^2$$

$$(P - n^2)^2 = 0$$

So the prize value is $P = n^2$ and each crewman's share is $P/n = n$, the number of men in the crew.

Now all we need to do is just use the first crewman's share to solve the problem, that is,

$$P/n = 1 + (P - 1)/100$$

$$n = 1 + (n^2 - 1)/100$$

$$n - 1 = (n^2 - 1)/100$$

$$1 = (n + 1)/100$$

$$n = 99$$

Thus there are 99 men in the crew and each man got $P/n = n^2/n = n = 99$ pounds. ($P = 99^2 = 9801$ pounds.)

It bothered me a bit that I did not use the shares of the other crewmen. I guess I should verify that the recursive steps all have equal shares. The following table shows what is happening.

Crewman	Share	Remainder
		$P = 9801 = (99 - 1)100 + 1$
First	$1 + 98 = 99$	$P_1 = P - 99 = 9702 = (99 - 2)100 + 2$
Second	$2 + 97 = 99$	$P_2 = P_1 - 99 = 9603 = (99 - 3)100 + 3$
…		$P_{k-1} = P_{k-2} - 99 = (99 - k)100 + k$
k^{th}	$k + (99 - k) = 99$	$P_k = P_{k-1} - 99$ $= [(99 - k)100 + k] - (100 - 1)$ $= (99 - (k + 1))100 + k + 1$

References

[1] Leybourn, Thomas, *The Mathematical Questions Proposed in the Ladies' Diary, and Their Original Answers, together with Some New Solutions, from Its Commencement in the Year 1704 to 1816*. J. Mawman, London (4 vols.). 1817. (https://archive.org/details/mathematicalque02leybgoog/page/n6, retrieved 4/27/2019)

[2] Swetz, Frank J., "'The Ladies Diary': A True Mathematical Treasure," *Convergence* (August 2018), DOI:10.4169/convergence20180827 (https://www.maa.org/press/periodicals/convergence/the-ladies-diary-a-true-mathematical-treasure, retrieved 4/27/2019)

[3] Albree, Joe, and Scott H. Brown "A valuable monument of mathematical genius: *The Ladies' Diary* (1704–1840)," *Historia Mathematica* 36 (2009) 10–47. Available online 24 December 2008 (https://www.sciencedirect.com/science/article/pii/S0315086008000967)