The Damaged Engine

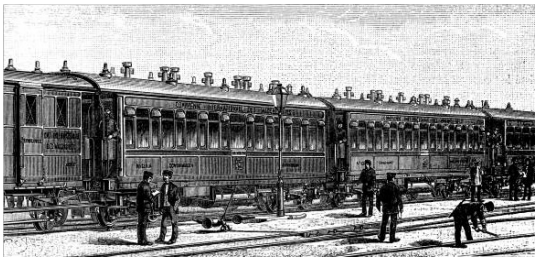
1 March 2019

Jim Stevenson

Yet another train problem from H. E. Dudeney.

69. THE DAMAGED ENGINE ([1] p.20)

We were going by train from Anglechester to Clinkerton, and an hour after starting an accident happened to the engine. We had to continue the journey at three-fifths of the former speed. It made us two hours late at Clinkerton, and the driver said that if only the accident had happened fifty miles farther on the train would have arrived forty minutes sooner. Can you tell from that statement just how far it is from Anglechester to Clinkerton?



Solution

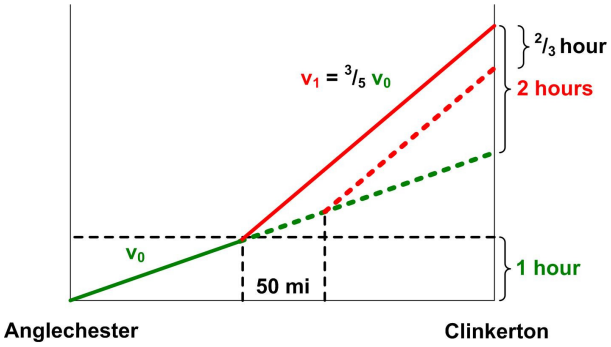


Figure 1 Problem Statement

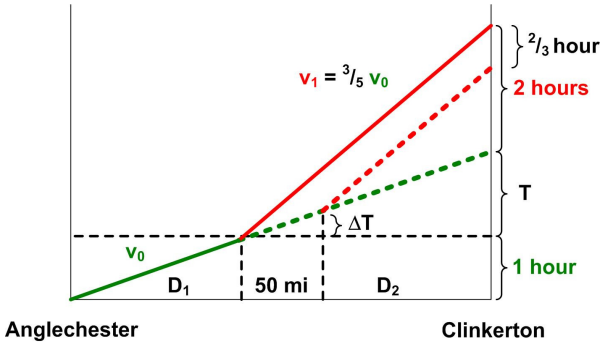


Figure 2 Problem Solution

Figure 1 shows the problem statement. Completing the labeling of the necessary variables in Figure 2 allows us to set up 6 equations in 6 unknowns. First, we consider the consequences of the initial speed v_0 of the train before it stopped. We have

$$v_0 \cdot 1 \text{ hr} = D_1 \text{ miles} \tag{1}$$

$$v_0 \cdot \Delta T \text{ hr} = 50 \text{ miles} \tag{2}$$

$$v_0 \cdot (T + 1) = D_1 + 50 + D_2 \quad \text{(total distance)} \tag{3}$$

$$v_1 \cdot (T + 2) = 50 + D_2 \tag{4}$$

$$v_1 \cdot (T + 2 - {}^2/_3 - \Delta T) = D_2 \tag{5}$$

$$v_1 = {}^3/_5\, v_0 \tag{6}$$

Equations (1) – (4) and (6) yield

$$v_0 \cdot (T + 1) = v_0 + {}^3/_5\, v_0 \cdot (T + 2)$$

Canceling v_0 yields $T = 3$ hours. Equation (4) with $T = 3$ becomes

$$3v_0 - 50 = D_2$$

And equation (5) with equation (2) and T = 3 becomes

$$^{13}/_{5}\, v_0 - 30 = D_2$$

Eliminating D_2 between the two equations means $v_0 = 50$ mph. Therefore from equation (3) we get the total distance between Anglechester and Clinkerton is $v_0 \cdot (T + 1) = 50 \cdot 4 =$ 200 miles.

References

[1] Dudeney, Henry Ernest, *536 Puzzles & Curious Problems*, Edited By Martin Gardner, Charles Scribner's Sons, New York, 1967.