Missing Pages Puzzle

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Jim Stevenson

Setting aside my chagrin that the following problem was given to pre-university students, I initially found the problem to be among the daunting ones that offer little information for a solution. It also was a bit "inelegant" to my way of thinking, since it involved considering some separate cases. Still, the end result turned out to be unique and satisfying (Talwalkar's Note 2 was essential for a unique solution, since the problem as stated was ambiguous).

(https://mindyourdecisions.com/blog/2018/04/19/the-seemingly-impossible-missing-book-pages-puzzle-from-india/, retrieved 4/25/2018)

The Seemingly Impossible Missing Book Pages Puzzle From India

Presh Talwalkar, April 19, 2018

Kshitij from India sent me this problem from the 1994 India Regional Mathematics Olympiad.

"A leaf is torn from a paperback novel. The sum of the numbers on the remaining pages is 15000. What are the page numbers on the torn leaf?"

Note 1: a "leaf" means a single sheet of paper.

Note 2: the quoted problem is actual wording from the competition. But let me add an important detail: the book is numbered in the usual sequential way starting with the first page as page 1.

My Solution

My solution essentially followed Talwalkar's, except that I also proved it was unique. First, we note the standard result for a sum of consecutive N integers:

$$1 + 2 + 3 + \ldots + N = N(N+1)/2$$

So the problem says for some N

$$N(N+1)/2 = 15{,}000 + m$$

where m is the sum for the two page numbers on the missing leaf. Since the missing page numbers must be consecutive,

$$m = k + (k + 1) = 2k + 1$$

which is an odd number. Moreover, (this is where Note 2 comes in), k itself must be odd, since the first leaf (pair of pages) has pages 1 and 2, and 1 is odd.

So we are looking for the number of pages in the book N whose consecutive sum exceeds 15,000. To see where to begin, find an N such that

$$N(N+1)/2 = 15{,}000 \text{ or } N^2 + N = 30{,}000.$$

Then $N = (-1 + \sqrt{1+4 \times 30000})/2 \approx 100\sqrt{3} \approx 174$. Now consider the following table

N	1+2+ … +N	m = 2k + 1	k	k + 1
173	15051	51	**25**	**26**
174	15225	225	112 (not odd)	113
175	15400	400 (not odd)		
176	15576	576 (not odd)		
177	15753	753	376 (>177)	377 (>177)

We can see from the table that the only pair of missing pages that meets all the criteria is 25,26 out of a book of 173 pages (the last leaf of the book must have a blank back). A book of less than 173 pages will not add up to a number greater than 15000 and a book of more than 176 pages would only yield possible missing page numbers greater than the number of pages in the book. So 25,26 is the unique set of missing page numbers.

Talwalkar, as did I, initially arrived at 112,113 as the answer, and then realized that the pair had to begin with an odd page number, so that left 25,26. Talwalkar did not mention the elimination of any other possibilities.