Microwave Problem

(3 August 2011, rev 10 February 2018)

Jim Stevenson

As another instance of how mathematicians look at the world differently, here is the following meditation that consumed my mind the other day. We finally got a new microwave oven after something like 25 years. The old one did not have a carousel, whereas the new one does. The carousel consists of a glass plate that rests on a plastic bearing ring that has several small plastic vertical wheels or rollers that run around the bottom of the microwave and the bottom of the glass dish. The glass dish also has some glass "feet" or knobs in the bottom so that when the plate is put on a counter, it can rest on those feet. As I watched the carrousel go around, I noticed the feet, which are located about six inches from the center but not quite as far out as the plastic roller ring underneath, were moving around much faster than the underlying roller ring. (See Figure 1)

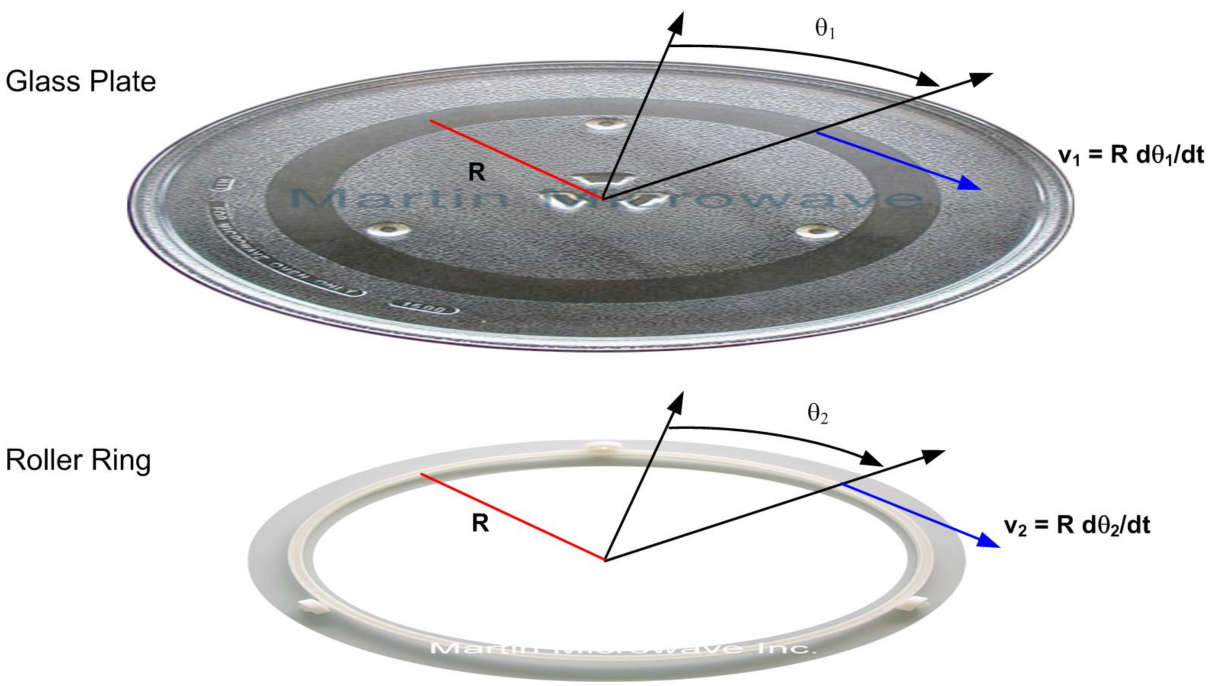


Figure 1 Glass Plate on Roller Ring Assembly with Annotations

So the obvious question is why? Further, what is the mathematical relationship between the speed of rotation of the glass dish ($d\theta_1/dt$) and the underlying roller ring ($d\theta_2/dt$)? I am guessing it has something to do with the radius of the bearing ring and the radii of the little roller wheels attached to it. Anyway, it is a fun question and one I have never seen addressed in any of the books about "practical" math questions I read.

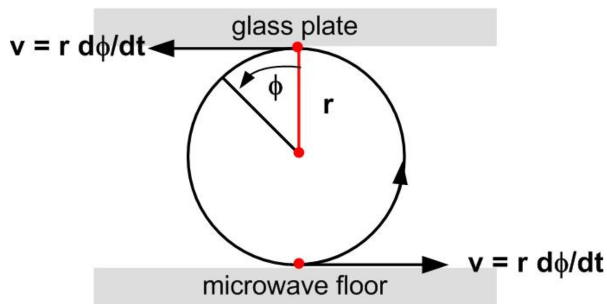
Give some thought before turning the page to see what I figured out.

Answer:

The glass plate rotates twice as fast at the roller ring. Surprisingly, this is independent of the size of the rollers, nor does it depend on the rollers' radial distance from the center.

Solution:

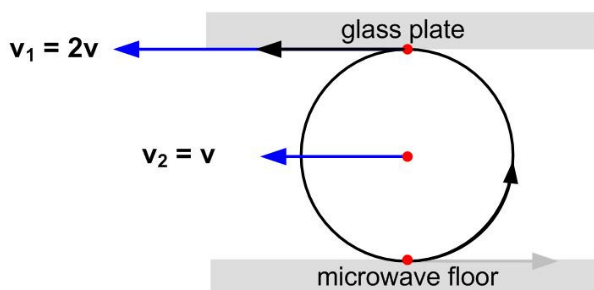
Consider a roller wheel turning between the plate and the floor of the microwave – from the roller's viewpoint, that is, with its center axis fixed (Figure 2). It rotates with an angular speed ($d\phi/dt$) determined by the movement of the glass plate above it. The tangential speed (v) of a point moving on the edge of the wheel at a distance r from the center is $v = r\, d\phi/dt$. This tangential speed is the same for all points on the roller wheel edge. The red dot at the top of the roller in the Figure 2 represents where the glass plate touches the roller, and so the glass plate has the same leftward speed v. Similarly, the red dot at the bottom of the roller represents where the microwave floor touches the roller, and so the microwave floor has the same rightward speed of v, relative to the fixed center of the roller.



Roller motion relative to roller (center fixed).

Figure 2 Roller Wheel Motion Relative to Center of Roller

Now consider the same turning roller wheel from the point of view of the floor of the microwave (Figure 3). This is equivalent to canceling the rightward movement of the microwave floor by adding the speed v to the left of each point in Figure 2. This means (the center of) the wheel will move to the left with a speed v, the top point touching the glass plate (and so the plate) will move to the left with a speed equal to v + v or 2v, and the microwave floor will "move" with a speed v – v = 0.



Roller motion relative to microwave floor (center moving).
(Add left directed v to the three (red) points.)

Figure 3 Roller Wheel Motion Along Microwave Floor

If the wheel is straight up and down, then these three points are all a distance R, say, from the center of the ring and plate. This means the radius R in the tangential speeds v_1 and v_2 in Figure 1 cancel (Figure 4), leaving the rotation rate of the glass plate twice that of the plastic roller ring underneath.

$$\therefore\ v_1 = 2\, v_2 \ \Rightarrow\ d\theta_1/dt = 2\, d\theta_2/dt$$

Figure 4 Plate Rotates Twice as Fast as Roller Ring

Note: This doubling of the rotation rate of the glass plate relative to the roller ring does not depend on the radius of the roller wheel r, nor on the radius of the roller ring R. Of course the roller wheel will rotate at different speeds depending on its diameter, but the doubling effect still holds.

Update 2/10/2018: It looks like this problem has shown up before in other guises. One of which I came across at the *Futility Closet* website:

Rock and Roll

(1 August 2009)

(https://www.futilitycloset.com/2009/08/01/rock-and-roll-2/, retrieved 2/10/2018)

Worshipful natives are rolling a giant statue of me across their island. The statue rests on a slab, which rests on rollers that have a circumference of 1 meter each. How far forward will the slab have moved when the rollers have made 1 revolution?

Answer

The slab will have moved forward 2 meters, not 1. If the slab were removed, 1 turn would advance the rollers 1 meter. If the rollers were held in place, 1 turn would advance the slab 1 meter. Combining the two motions means that 1 turn advances the slab 2 meters.