The Keyhole Problem

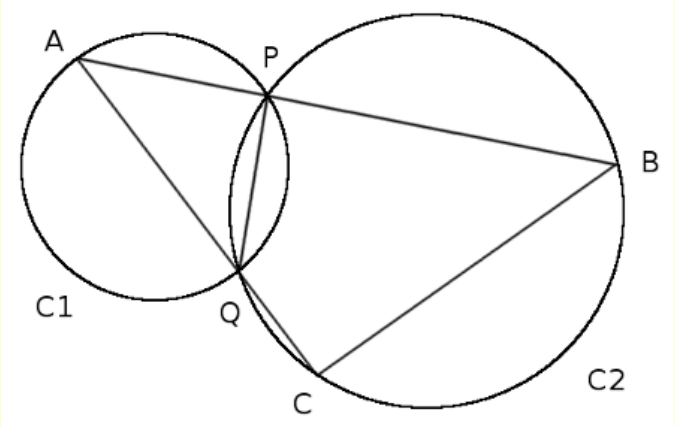
5 February 2016

Jim Stevenson

The following problem was found at the Futility Closet website:

(http://www.futilitycloset.com/2015/12/23/the-keyhole/, retrieved 2/5/16)

The Keyhole (23 December 2015)



Draw circles C1 and C2 with the common chord PQ. Now choose a point A on the arc of C1 that's outside of C2 and project it through P to B and through Q to C.

Surprisingly, the length of BC remains the same no matter where A is chosen on its arc of C1.

Solution:

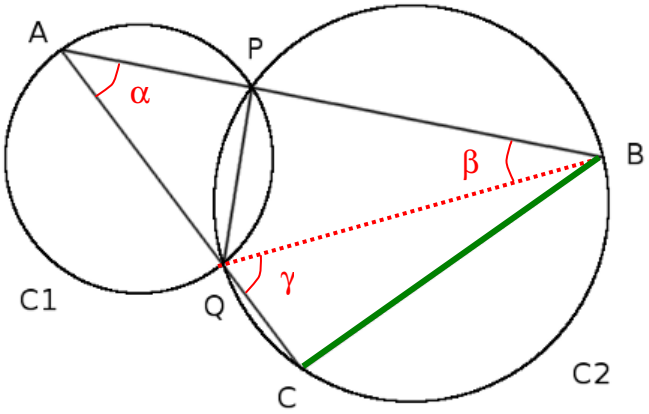


Figure 1 Parameterized Figure

As shown in Figure 1, add the line from Q to B. Let $\alpha = \angle PAQ$, $\beta = \angle PBQ$, and $\gamma = \angle BQC$. Since angle α subtends the same arc of the small circle no matter where A is positioned, its value is always the same. Similarly for angle β at B on the larger circle. Therefore, $\gamma = \alpha + \beta$ is also constant for all positions of A. And so the arc BC, and therefore the chord BC, are also the same length for all positions of A.