Corner Reflectors

(15 April 2016)

Jim Stevenson

I came across the following entry in the Futility Closet website that cried out for justification.

FUTILITY CLOSET

An idler's miscellany of compendious amusements

(http://www.futilitycloset.com/2016/04/13/corner-reflectors/, retrieved 4/14/2016)

Corner Reflectors (13 April 2016)

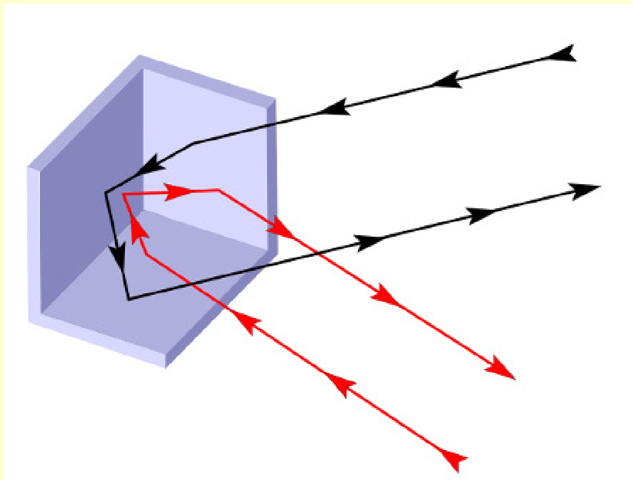

Image: Wikimedia Commons

An arrangement of three mutually perpendicular planes, like those in the corner of a cube, have a pleasing property: **They'll reflect a ray of light back in the direction that it came from.** This happy fact is exploited in a variety of technologies, from laser resonators to radar reflectors; the taillights on cars and bicycles contain arrays of tiny corner reflectors.

"A more dramatic application is to reflect laser rays from the Moon, where many such devices have been in place since the 1969 Apollo mission, which sent men to the Moon for the first time," note mathematicians Juan A. Acebrón and Renato Spigler. "Among other things, the Earth-Moon distance can be measured by firing a laser beam from the Earth to the Moon, and measuring the travel time it takes for the beam to reflect back. This has allowed an estimate of the distance to within an accuracy of 3 cm."

(Juan A. Acebrón and Renato Spigler, "The Magic Mirror Property of the Cube Corner," *Mathematics Magazine* 78:4 [October 2005], 308-311.)

So the question is, why is this reflection property true?

First, we need to make clear how a ray of light reflects off a planar surface. Figure 1 shows a beam of light (black line) reflecting off a (blue) plane. The figure also shows a normal (dashed) line perpendicular to the plane. The incoming, incident ray and the normal line define a (pink) plane perpendicular to the blue plane and containing the light beam. The reflected beam remains in the red plane with an angle of reflection θ equal to the angle of incidence. Other definitions define the angles of incidence and reflection relative to the normal line where they are also equal.

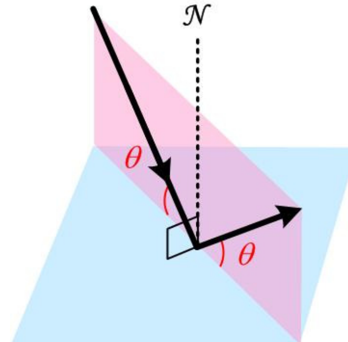

Figure 1 Angle of Incidence = Angle of Reflection

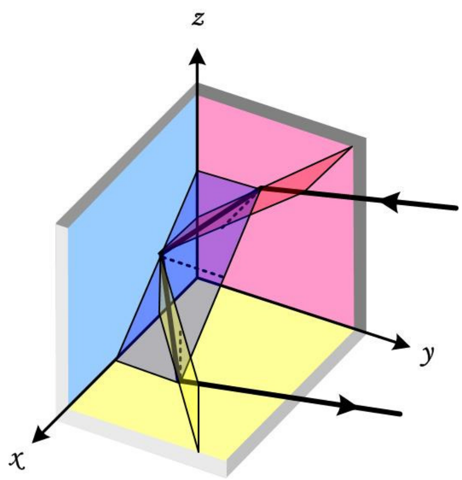

Figure 2 Light Beam and a Corner Reflector

Figure 2 shows a light beam hitting a corner reflector as in the Futility Closet picture. We are going to see what the beam looks like projected along each of the normals to the walls.

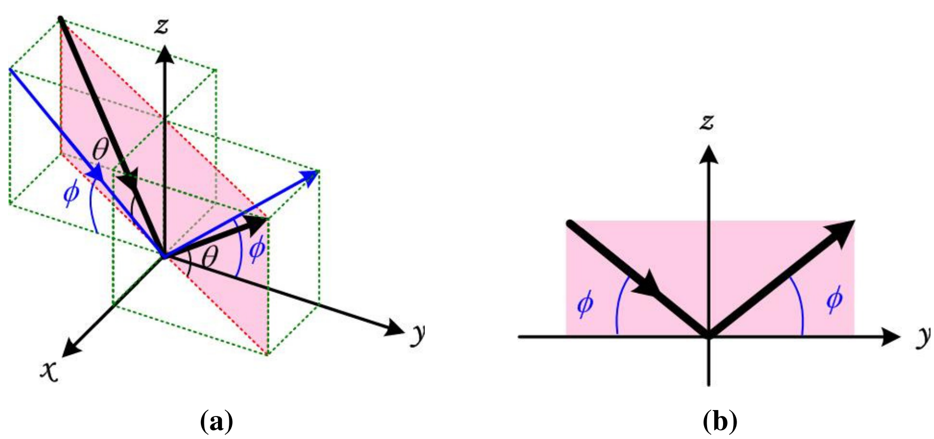

Figure 3 Projection of Light Beam Along x-axis Onto yz-plane

First we consider one normal direction. Figure 3 (a) shows a (black) reflected light beam projected along the x-axis onto the blue lines in the yz-pane. It is easy to see corresponding angles ϕ the blue lines make with the y-axis are also equal. Figure 3 (b) shows the view of the light beam looking down the x-axis toward the yz-plane. We get a similar projection along the y-axis onto the xz-plane with equal angles. The projection of the beam along the z-axis onto the xy-plane just yields a straight line with a given angle with respect to the x-axis. Notice that any two items from the collection of the straight line and the two projection angles completely determine the direction of the light beam (since each pair determines two planes that intersect along the light beam).

Now consider the three projections of the light beam in Figure 2 along the x-, y-, and z-axes. The results are shown in Figure 4. In the three cases the projections of the incidence angles (or their normal complements) α, β, γ equal the projections of the reflected angles. This means that the direction of the outgoing beam is the same as (parallel to) the direction of the incoming beam, which is what we wanted to show.

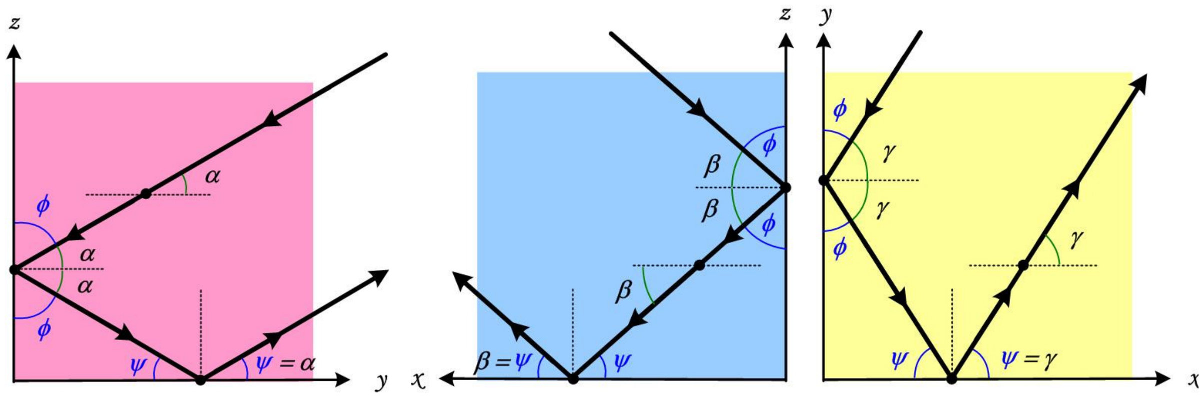

Figure 4 Projections of Light Beam Along the x-, y-, and z-axes

Notice from Figure 4 that the reason the reflection property holds is that each projection reveals only two reflections, which essentially cancel each other. The other condition is that the two sides where the reflections occur must be at right angles to each other. If we added another wall creating a second corner, then some of the light rays would be reflected off a third side, causing an odd number of reflections and so exit in a different direction from the entering ray. And if we had a set of walls meeting at obtuse angles, such as the sides of a dodecahedron (Figure 5), then the reflection property would also fail.

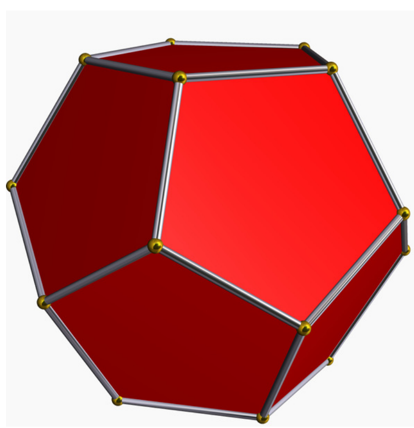

Figure 5 Regular Dodecahedron