Ant Problem

(19 January 2016)

Jim Stevenson

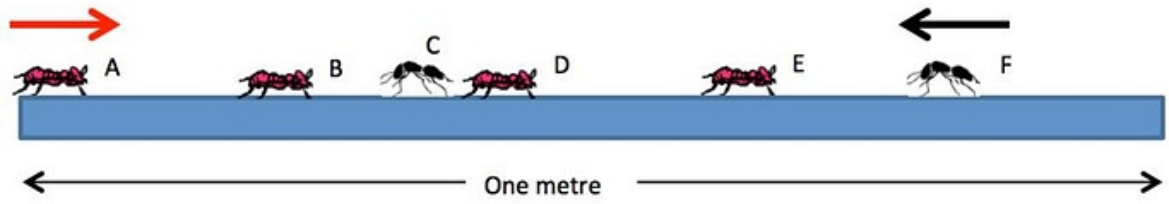
Alex Bellos had the following Monday problem:

Can You Solve It? The Ants On A Stick Puzzle[1]

Alex Bellos, 18 January 2016

Hello guzzlers.

Many classic puzzles involve animals, such as the ones about a bird flying between trains,[2] a fox and a goose crossing a river[3] and four dogs chasing each other.[4] A recent addition to the genre involves ants on a stick. Today's variation is by maths author Rob Eastaway,[5] about whom more later. Okay, to today's puzzle:



A deck of ants: A, B, D and E are moving left to right. C and F from right to left. Illustration: Rob Eastaway

Four red ants and two black ants are walking along the edge of a one metre stick. The four red ants, called Alf, Bert, Derek and Ethel, are all walking from left to right as we look at the diagram, and the two black ants, Charlie and Freda, are walking from right to left.

The ants always walk at exactly one centimetre per second. Whenever they bump into another ant, they immediately turn around and walk in the other direction. And whenever they get to the end of a stick, they fall off.

Alf starts at the left hand end of the stick, while Bert starts 20.2 cm from the left, Derek is at 38.7cm, Ethel is at 64.9cm and Freda is at 81.8cm. Charlie's position is not known — all we know is that he starts somewhere between Bert and Derek.

So here is the puzzle: ***Which ant is the last to fall off the stick? And how long will it be before he or she does fall off?***

I'll be back later today with the solution. Thanks Rob Eastaway for the puzzle. Rob's new book *Maths on the Go: 101 Ways to Play with Maths*[6] was out last week.

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1 http://www.theguardian.com/science/2016/jan/18/can-you-solve-it-the-ants-on-a-stick-puzzle, retrieved 1/22/16

2 http://math.stackexchange.com/questions/784888/apparent-paradox-for-the-bird-traveling-between-two-trains-puzzle

3 https://en.wikipedia.org/wiki/River_crossing_puzzle

4 http://www.mytechinterviews.com/chasing-dogs

5 http://www.amazon.co.uk/gp/product/0224101625/ref=as_li_tl?ie=UTF8&camp=1634&creative=19450&creativeASIN=0224101625&linkCode=as2&tag=alebel-21

6 http://www.robeastaway.com/blog/maths-play

The Answer

Charlie falls off last after exactly 100 seconds.

The workings: Seasoned readers of this column will have immediately realised that the arbitrary-sounding distances — 20.2cm, 38.7cm, etc — were a decoy. These lengths were a sign that you needed to think laterally about the puzzle. No half-decent puzzle is going to require algebra or arithmetic with numbers like 20.2 and 38.7. Certainly not here!

Of course, it would be possible to work out the answer by calculating the positions but this will be messy. (Full marks for effort for all of you who tried to do it this way.)

The piece of insight that makes this an elegant puzzle is this: imagine two ants colliding and turning around. If you blur your eyes, this is equivalent to those two ants walking past each other. In other words, this puzzle can be treated as if it is six ants each on their own track walking to the end of the stick. Since Alf starts at the left end of the stick walking to the right, one of these 'blurred' ants will walk the maximum possible distance — the entire length of the stick — and then fall off. So, it will take 100 seconds — 1m at 1 cm per sec — until the last ant drops.

Or, as reader William Guest[7] elegantly wrote in a comment, imagine that each ant has a leaf, and on each collision they exchange leaves. The leaves will be moving at 1cm per sec in a unique direction. The leaf that will take the longest to fall off the edge is the one that starts with Alf.

Now to the identity of the ant that falls last.

All we need to remember is that the order of the ants cannot change since they can't walk past each other. At the start, four ants are walking to the right. So four ants will drop off the right hand end. So the ant that is positioned fourth to the right will fall last. Stand up Charlie, that's you.

The fact that we don't know his position to start with is irrelevant.

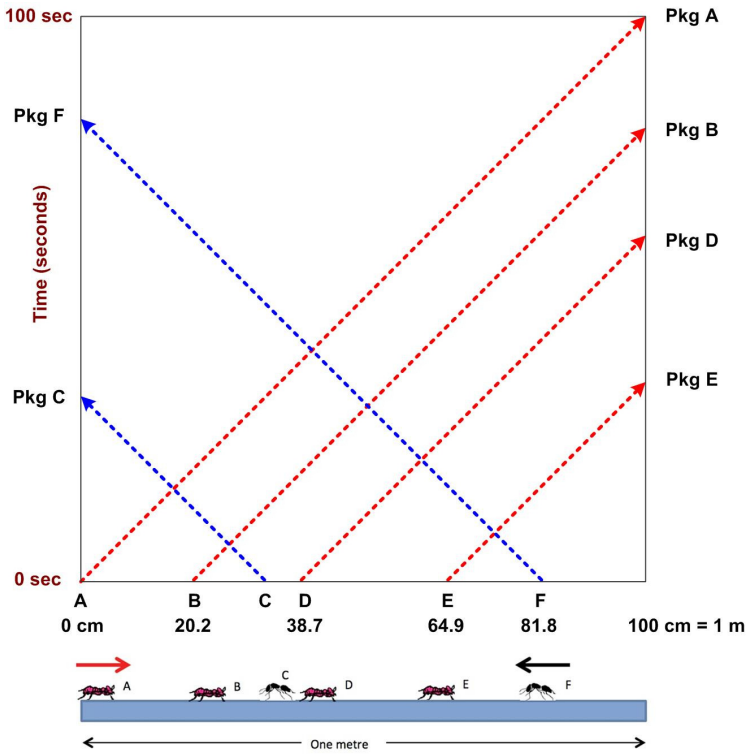
My Solution

My solution essentially agrees with Bellos's and those of his contributors, though I have provided more visual support. It is actually easier to answer the puzzle (*Which ant is the last to fall off the stick? And how long will it be before he or she does fall off?*) in reverse order, as did Bellos.

To show how long before the last ant falls off, I imagined the same idea as William Guest above, namely, imagine each ant carrying a package. When two ants meet, they exchange packages. Because these two ants will then reverse direction, the exchanged packages will still continue on in their original direction. The packages move as fast as the ants carrying them, namely, 1 cm/sec. So even though the ants are ricocheting back and forth, the packages are moving in a constant direction and constant speed of 1 cm/sec. So the package originally carried by ant A, designated Pkg A, has to traverse the entire length of the meter stick and so will take 100 sec to reach the other end (carried by some other ant). This will be the longest trip and that ant will be the last one to fall off the stick (after 100 sec).

The following diagram illustrates the traveling package idea with the horizontal axis representing the meter stick and the vertical axis time in seconds. So Pkg A starts out at the left edge and proceeds to the right edge at 1 cm/sec, arriving after 100 seconds.

[7] https://profile.theguardian.com/user/id/1352616



Now to find out what ants finally deliver the packages, and in particular Pkg A, consider the next figure. When each ant encounters another, it reverses direction and shows a zig-zag pattern in the space-time diagram. And so we see it is Ant C, Charlie, who carries Pkg A last.